Electromagnetic Modeling of Superconducting High Field Magnets

November 2021

Dr. Edgar Berrospe Juárez

POLYTECHNIQUE MONTRÉAL

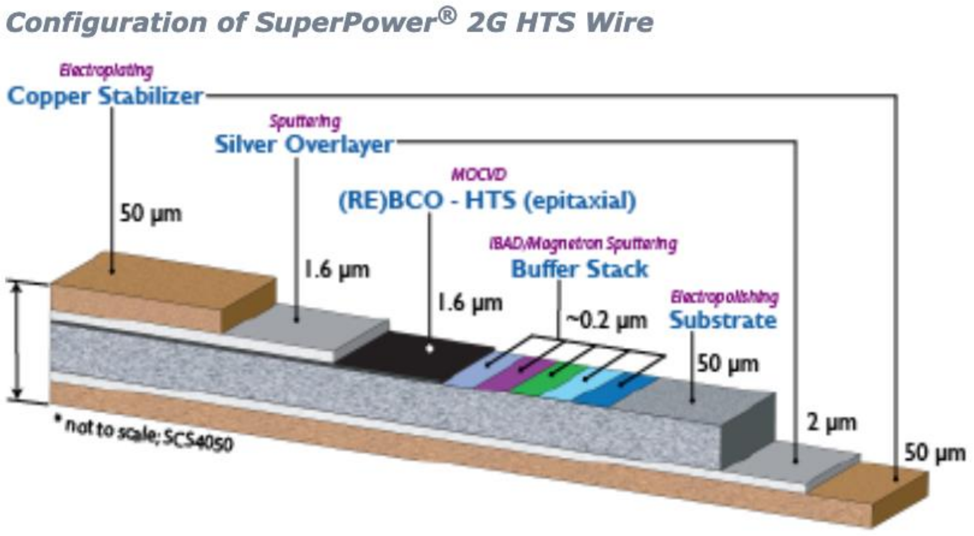
INSTITUTO DE INGENIERÍA UNAM

UNIVERSIDAD NACIONAL AUTONOMA D MEXICO

KIT Karlsruhe Institute of Technology

Objectives

- Develop simulation tools to estimate current density, magnetic field and hysteresis losses in large-scale superconductor systems, i.e., systems made up of from hundreds to thousands of turns of REBCO tapes.
- Compare simulation results with experimental data:
    - 32 T all-superconducting magnet (NHMFL, Florida).
- Develop adequate operation strategies for existing and future systems.



Analytical Results

H formulation

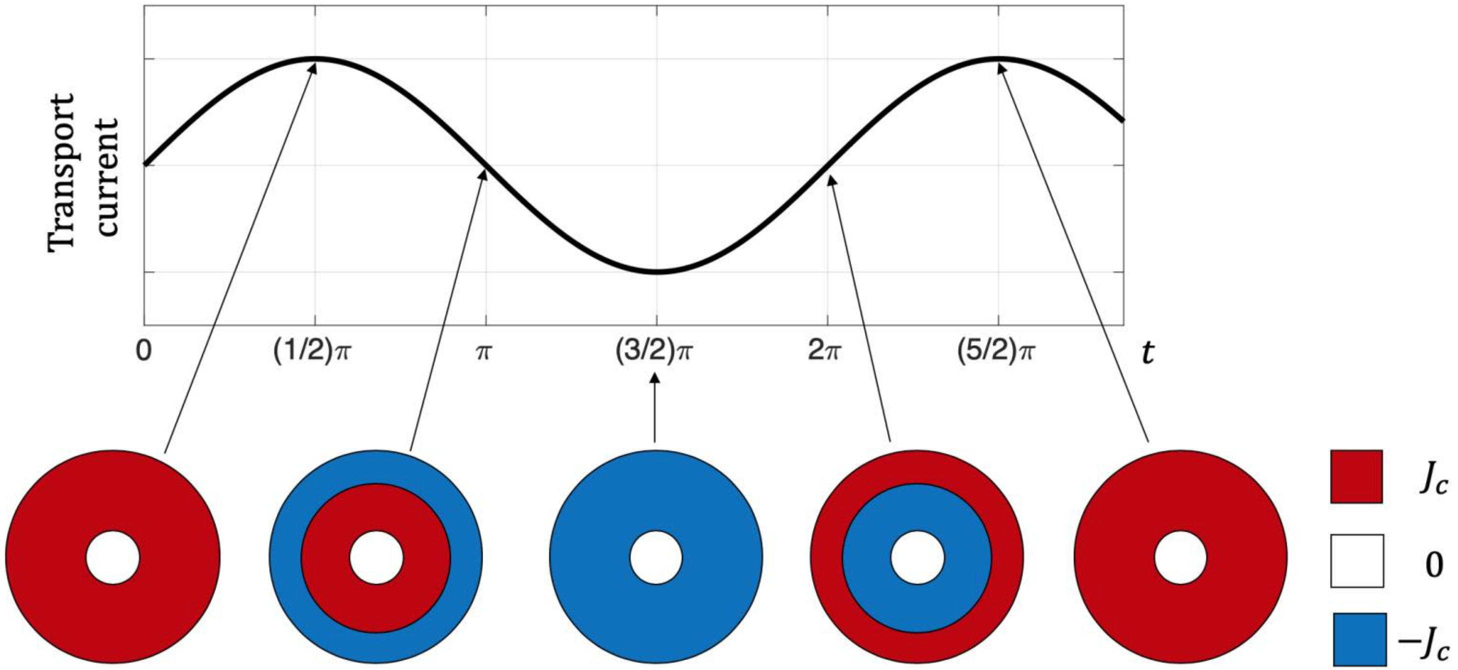
- Homogenization and *H* formultaion
- Multi-scaling and *H* formulation

T-*A* formulation

- Multi-scaling and *T*-*A* formulation
- Homogenization and *T*-*A* formultaion

Critical-State Model

- The critical-state model (CSM) is a phenomenological description introduced in [*Bean, 1962*] to describe the magnetic hysteresis of type-II superconductors.
- The CSM states that, no current flows in the regions that are not previously penetrated by the magnetic field.
- The change in the magnetic field produces the appearance of electric field $\mathbf{E}$.
- Any $\mathbf{E}$ value, however small, will induce the critical current density J_c, to flow.



Analytical Results

- [*Norris, 1969*] Losses produced by transport currents.
- [*Brandt, 1994*] Losses and current density produced by external fields.

Calculation of hysteresis losses in hard superconductors carrying ac: isolated conductors and edges of thin sheets

W. T. NORRIS

Central Electricity Research Laboratories of the Central Electricity Generating Board, Leatherhead, Surrey

MS. received 19*th May* 1969, *in revised form* 28*th November* 1969

Abstract. Two methods of calculating hysteresis losses in hard superconductors are described. The London model is assumed in which the critical current density is taken independent of magnetic field. Losses in isolated wires of different cross section are considered but it is found that losses for solid wires vary by at most a factor of 3 for different shaped wires of the same current-carrying capacity. The loss at saturation current is usually $0{\cdot}4$–$0{\cdot}6\ I_c{}^2\mu_0/\pi$.

Losses at the edges of thin sheets are also calculated and a fourth-power dependence on current (for low current) is found. Three systems are examined: a slit parallel to the current in a wide sheet ($L_c \simeq \mu_0 j^2 g^2 \pi^3 F^4/24$), one pair of the edges of two wide strips set back-to-back and carrying antiparallel currents ($L_c \simeq \mu_0 \pi j^2 s^2 F^4/6$) and a long thin wall parallel to the current flow on a wide sheet ($L_c \simeq \mu_0 \pi^3 j^2 a^2 F^4/3$). L_c is the loss per cycle per unit length, F is the current peak as a fraction of saturation current, g the width of the slit, s the spacing of strips, a the height of the asperity and j the critical current density per unit width. All in MKS units.

1. Introduction

Any theoretical estimation of ac losses in a superconductor is unlikely to accord with measurements very accurately. The materials are variable and we are still uncertain what are the important physical qualities which influence losses. This is particularly so for type I or surface superconductors, run below their first critical field.

For worked type II materials, the so-called hard superconductors (HSC), run at high currents, an approximate method of calculation due to London (1963) is available. An idealized behaviour is assumed. The superconductors can carry current up to some maximum critical current density and then become resistive. The resistance is assumed to rise very steeply as the current tries to increase above the critical value and the resistance

PHYSICAL REVIEW B VOLUME 49, NUMBER 13 1 APRIL 1994-I

Thin superconductors in a perpendicular magnetic ac field: General formulation and strip geometry

Ernst Helmut Brandt

Max-Planck-Institut für Metallforschung, Institut für Physik, D-70506 Stuttgart, Germany

(Received 16 November 1993)

The sheet current, electric field, and penetrating magnetic field in response to an applied perpendicular ac magnetic field are calculated for a thin type-II superconducting strip characterized completely by its sheet resistivity, which may be either nonlinear and frequency independent or linear, complex, and frequency dependent. The general formulation is given for the linear or nonlinear response of a strip and a circular disk in perpendicular time-varying magnetic field. An elegant and rapid numerical method is presented which solves this, in general, nonlinear one-dimensional integrodifferential equation with high precision on a personal computer and which accounts for the facts that the integral kernel has a logarithmic singularity and the sheet current for nearly ideal shielding (occurring at short times or high frequencies or for strong pinning of flux lines) has a one-over-square-root singularity near the specimen edges. As examples the linear Ohmic response of the strip to a sudden change of the applied field and to an ac field are given; Ohmic response is realized during flux flow or thermally activated flux flow. The complex magnetic susceptibility and the ac losses of the Ohmic strip are computed and approximated by simple expressions. This work completes the calculation of dissipation peaks in vibrating superconductors caused by various diffusion modes of the flux lines.

I. INTRODUCTION

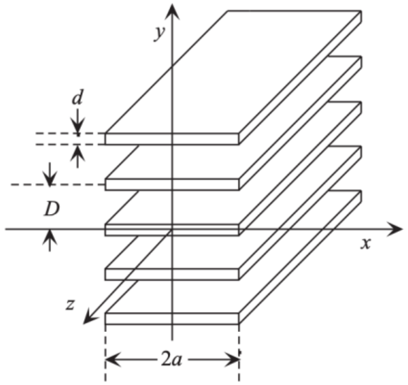
The discovery of high-T_c superconductors (HTSC's) has revived the interest in experimental methods which measure the electromagnetic response of these extreme type-II superconductors in dc and ac magnetic fields $H_a(t)$. Most of these experiments are performed in *perpendicular geometry*, in which a thin ceramic or monocrystalline platelet or film is exposed to a magnetic field that has a component perpendicular to the specimen. This geometry gives a larger response since it generates a much a larger magnetic moment per unit volume than the longitudinal geometry. Two examples may illustrate this, in which we consider the maximum negative magnetic moment $-m$ achieved during complete flux expulsion, e.g., in the Meissner state of superconductors, a field caused by the magnetization of all other volume elements. These demagnetization effects can be described by a demagnetization factor N ($0 \le N \le 1$) if the specimen has the shape of an ellipsoid and if the magnetic response of the material is linear.[1] Both requirements are in practice never satisfied in experiments on HTSC's. In spite of this, demagnetization corrections are usually applied by approximating disks or strips with rectangular cross section by ellipsoids with half axes a and b, yielding demagnetizing factors $N = 1 - \pi b/2a$ for thin disks and $N = 1 - b/a$ for thin strips in perpendicular fields ($b \ll a$). The field at the equator of the disk or edge of the strip is then enhanced by a large factor $1/(1-N)$; the same enhancement factor applies to the homogeneous effective magnetic field experienced by each volume element and to the magnetic moment as compared to the longitudinal geometry, which exhibits no demagnetization effects.

Analytical Results

- [*Norris, 1969*] Losses produced by transport currents.
- [*Brandt, 1994*] Losses and current density produced by external fields.

  - Just single conductors are analyzed.
  - The critical-state model is assumed.
  - Constant J_c is assumed.
  - Simultaneous transport currents and external fields are not considered.

- There also exist analytical results that assume infinite tape's stacks
  - [*Mawatari, 1996*].
  - [*Clem, 2008*].



Tape's stack, [Clem, 2008].

H Formulation

H Formulation

- The use of the Finite Element Method and the *H* formulation was presented in [*Brambilla et al., 2006*] and [*Hong et al., 2006*].
- This strategy allows the overcoming of the limitations of the analytical methods and of the formulations using potential vectors.

INSTITUTE OF PHYSICS PUBLISHING SUPERCONDUCTOR SCIENCE AND TECHNOLOGY
Supercond. Sci. Technol. **20** (2007) 16–24 doi:10.1088/0953-2048/20/1/004

Development of an edge-element model for AC loss computation of high-temperature superconductors

Roberto Brambilla[1], Francesco Grilli[2,3] and Luciano Martini[1]

[1] CESI Ricerca, Via Rubattino 54, 20134 Milano, Italy
[2] Superconductivity Technology Center, Los Alamos National Laboratory, Los Alamos, NM 87545, USA

E-mail: fgrilli@lanl.gov

Received 22 June 2006, in final form 26 October 2006
Published 20 November 2006
Online at stacks.iop.org/SUST/20/16

Abstract
This paper presents a new numerical model for computing the current density, field distributions and AC losses in superconductors. The model, based on the direct magnetic field **H** formulation without the use of vector and scalar potentials (which are used in conventional formulations), relies on first-order edge finite elements. These elements are by construction curl conforming and therefore suitable to satisfy the continuity of the tangential component of magnetic field across adjacent elements, with no need for explicitly imposing

INSTITUTE OF PHYSICS PUBLISHING SUPERCONDUCTOR SCIENCE AND TECHNOLOGY
Supercond. Sci. Technol. **19** (2006) 1246–1252 doi:10.1088/0953-2048/19/12/004

Numerical solution of critical state in superconductivity by finite element software

Z Hong, A M Campbell and T A Coombs

Cambridge University Engineering Department, Trumpington Street, Cambridge CB2 1PZ, UK

E-mail: zh223@cam.ac.uk

Received 24 August 2006, in final form 5 October 2006
Published 23 October 2006
Online at stacks.iop.org/SUST/19/1246

Abstract
A numerical method is proposed to analyse the electromagnetic behaviour of systems including high-temperature superconductors (HTSCs) in time-varying external fields and superconducting cables carrying AC transport current. The *E*–*J* constitutive law together with an H-formulation is used to calculate the current distribution and electromagnetic fields in HTSCs, and the magnetization of HTSCs; then the forces in the interaction between the electromagnet and the superconductor and the AC loss of the superconducting cable can be obtained. This numerical method is based on

H Formulation

- The use of the Finite Element Method and the *H* formulation was presented in [*Brambilla et al., 2006*] and [*Hong et al., 2006*].
- This strategy allows the overcoming of the limitations of the analytical methods and of the formulations using potential vectors
- The methods is validated against analytical and experimental results.

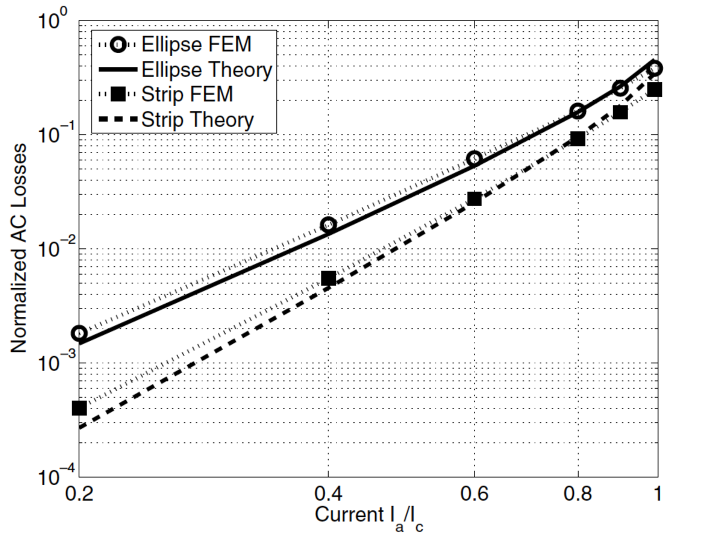


Figure 6. AC losses as a function of the transport current for an ellipse and a thin strip, compared to analytical predictions of Norris's formulae [13]. The losses are normalized by the factor $f I_c^2 \mu_0/\pi$.

Validation FEM vs Analytical, [Brambilla et al, 2006].

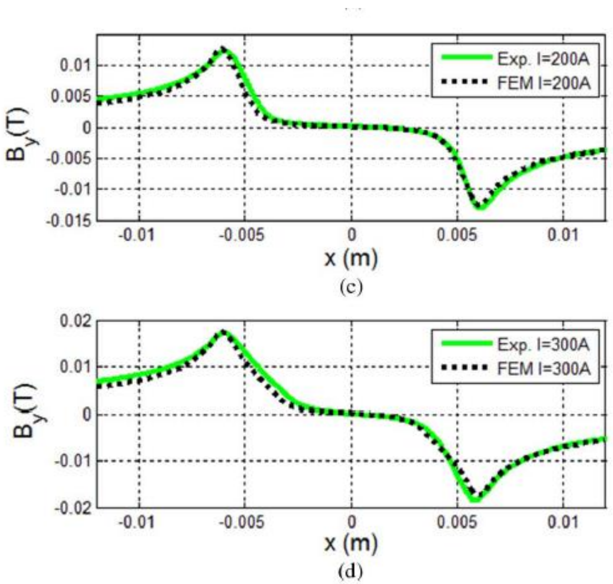


Fig. 15. Simulated and measured magnetic induction profiles for currents between 50 and 300 A during its rise.

Validation FEM vs experimental, [Sotelo et al., 2016].

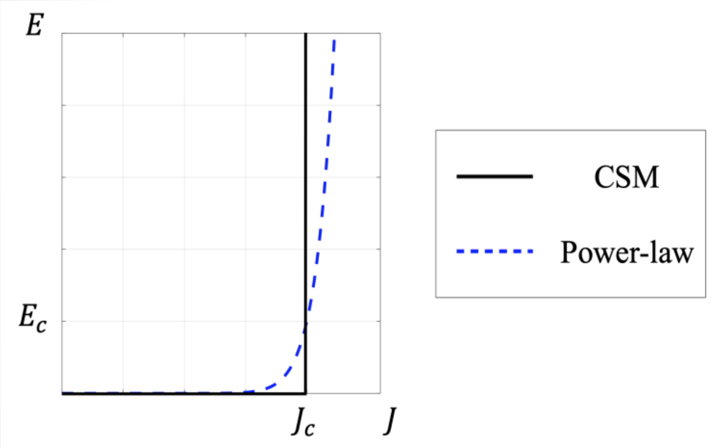
H Formulation of the Maxwell's Equations

$$\nabla \times \mathbf{E} = -\frac{\partial(\mathbf{B})}{\partial t}$$

H Formulation of the Maxwell's Equations

Non-linear resistivity

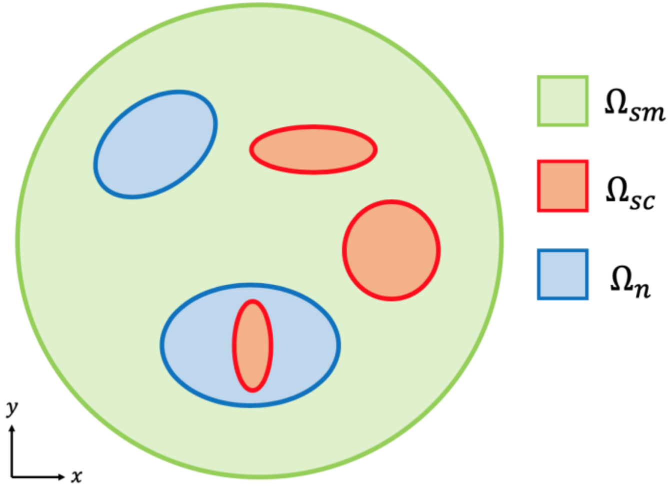
$$\rho = \frac{E_c}{J_c}\left|\frac{\boldsymbol{J}}{J_c}\right|^{n-1}$$



$$J_c = J_c(B, \theta)$$

$$\nabla \times \mathbf{E} = -\frac{\partial(\mathbf{B})}{\partial t}$$

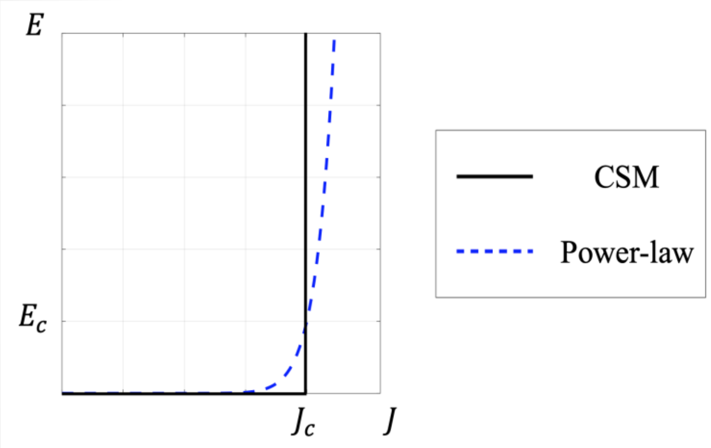
$$\mathbf{E} = \rho\mathbf{J} \qquad \mathbf{B} = \mu\mathbf{H}$$



H Formulation of the Maxwell's Equations

Non-linear resistivity

$$\rho = \frac{E_c}{J_c}\left|\frac{\boldsymbol{J}}{J_c}\right|^{n-1}$$

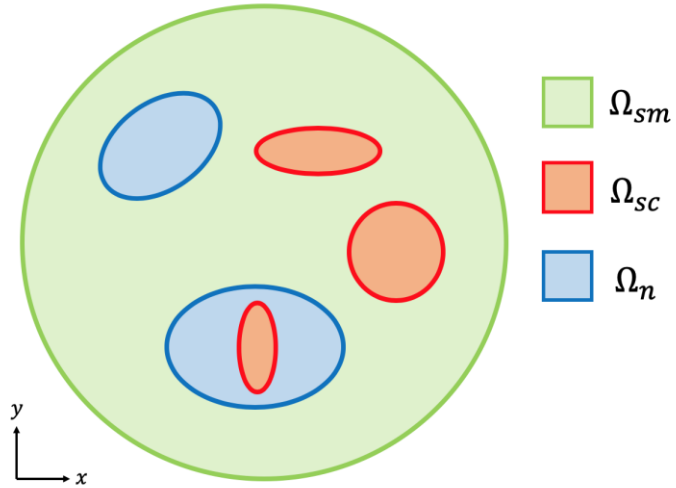


$$J_c = J_c(B, \theta)$$

$$\nabla \times \mathbf{E} = -\frac{\partial(\mathbf{B})}{\partial t}$$

$$\mathbf{E} = \rho\mathbf{J} \qquad \mathbf{B} = \mu\mathbf{H}$$

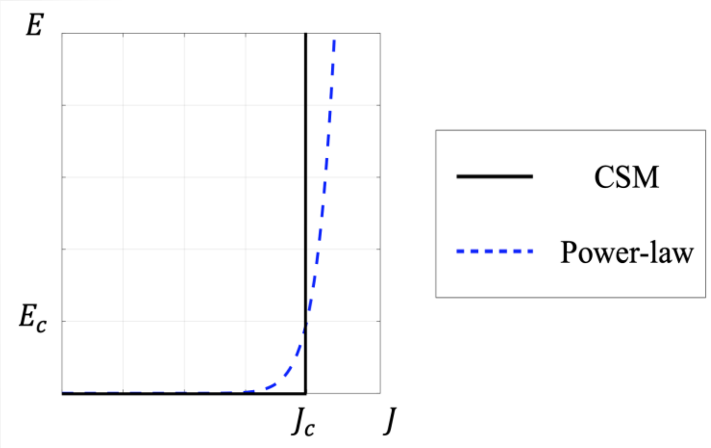
$$\nabla \times \rho\mathbf{J} = -\mu\frac{\partial(\mathbf{H})}{\partial t}$$



H Formulation of the Maxwell's Equations

Non-linear resistivity

$$\rho = \frac{E_c}{J_c}\left|\frac{\boldsymbol{J}}{J_c}\right|^{n-1}$$



$$J_c = J_c(B, \theta)$$

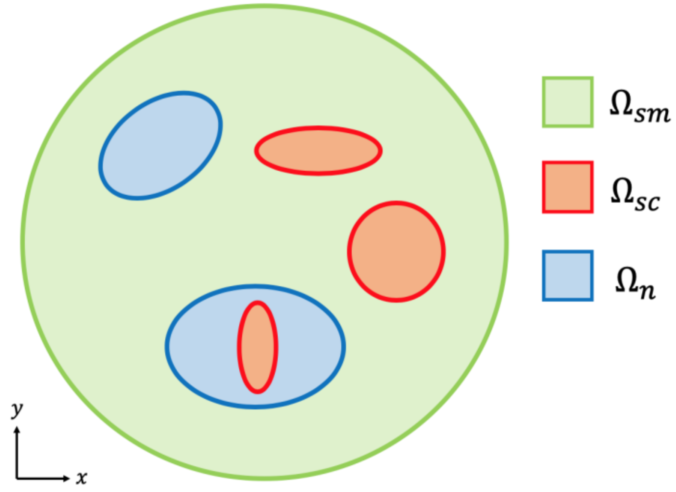
$$\nabla \times \mathbf{E} = -\frac{\partial(\mathbf{B})}{\partial t}$$

$$\mathbf{E} = \rho\mathbf{J} \qquad \mathbf{B} = \mu\mathbf{H}$$

$$\nabla \times \rho\mathbf{J} = -\mu\frac{\partial(\mathbf{H})}{\partial t}$$

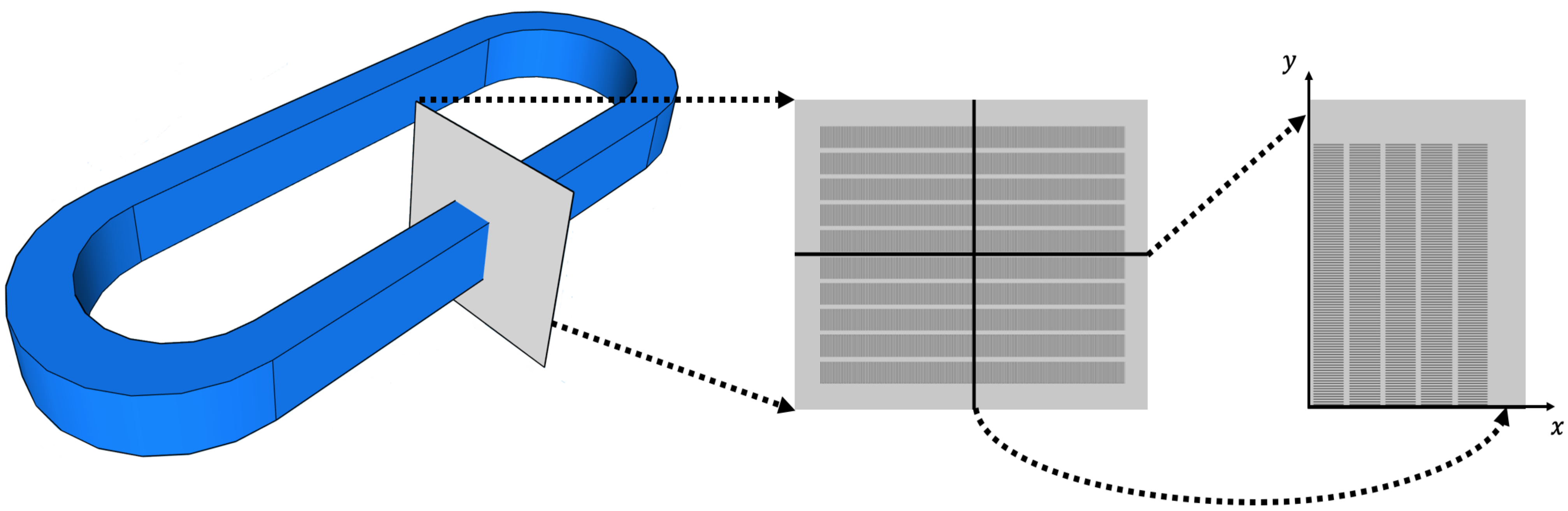
$$\nabla \times \mathbf{H} = \mathbf{J}$$

$$\nabla \times \rho\nabla \times \mathbf{H} = -\mu\frac{\partial(\mathbf{H})}{\partial t}$$



Case Study

- The case study is a racetrack coil made of HTS tapes.
- The symmeties of the system allow modeling ¼ of the cross-section of the coil.



Case Study – Reference Model (*H* Full Model)

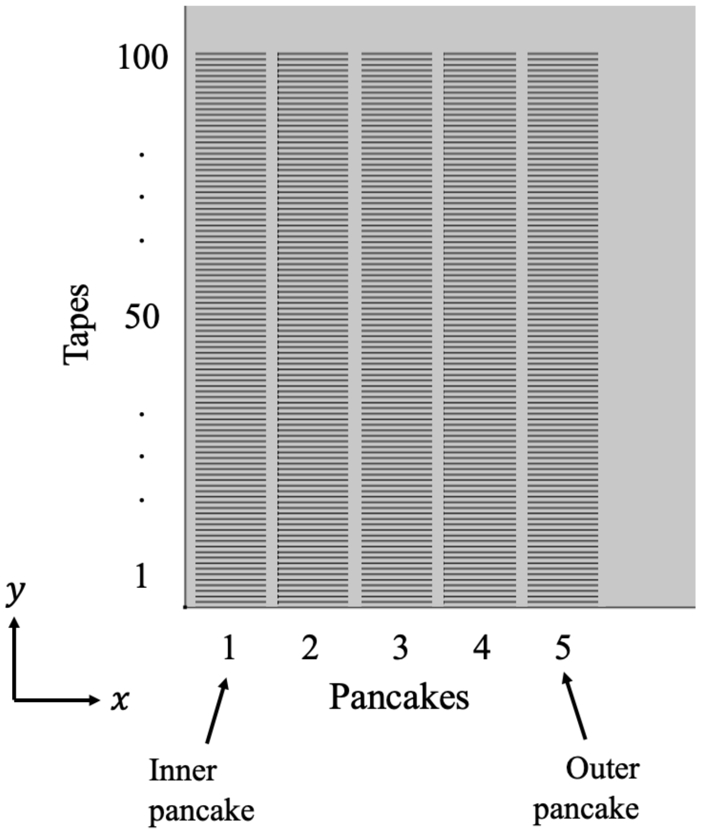


Table 1. Case study geometric parameters.

Parameter	Value
Pancakes	10
Turns per pancake	200
Unit cell width	4.45 mm
Unit cell thickness	293 μm
HTS layer width	4 mm
HTS layer thickness	1 μm

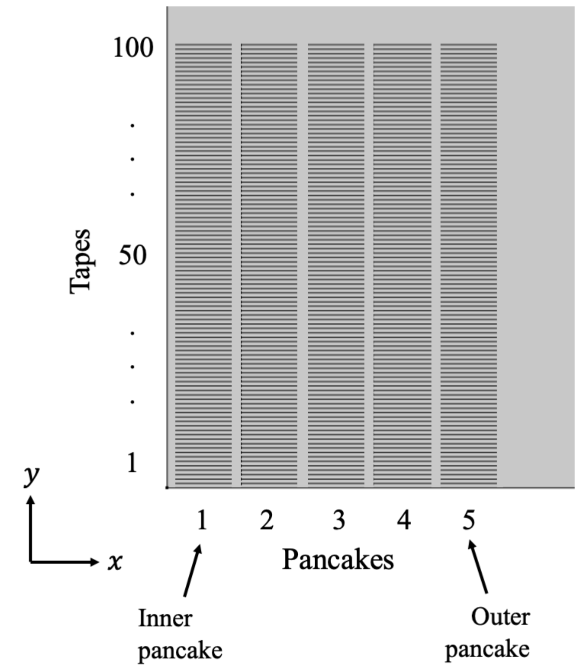
Table 2. Case study electromagnetic parameters.

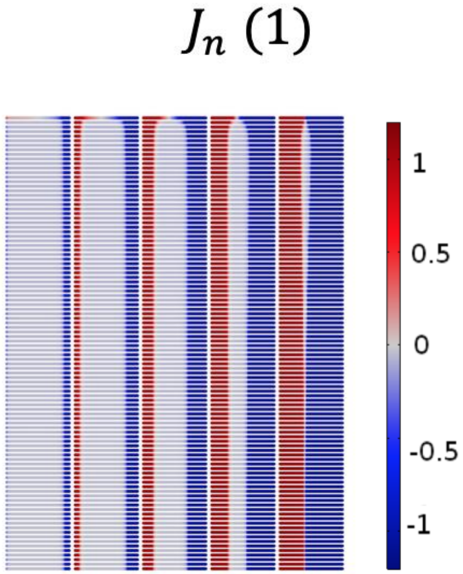
Parameter	Value
E_c	1×10^{-4} Vm^{-1}
n	38
J_{c0}	2.8×10^{10} Am^{-2}
B_0	0.04265 T
k	0.29515
α	0.7
Ω_{sm} resistivity ρ_{sm}	1 Ωm
μ_0	$4\pi \times 10^{-7}$ Hm^{-1}

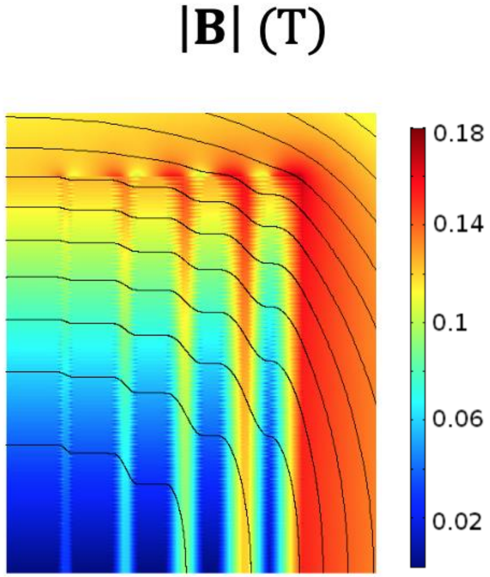
$$J_c(\boldsymbol{B}) = \frac{J_{c0}}{\left(1 + \frac{\sqrt{k^2 B_\parallel^2 + B_\perp^2}}{B_0}\right)^{\alpha}}$$

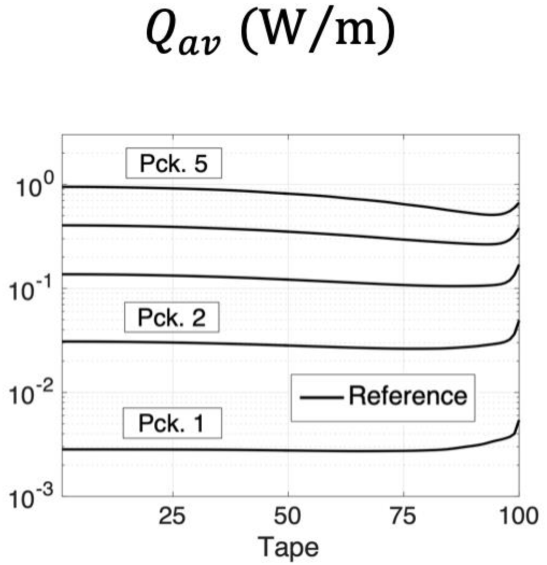
Case Study – Reference Model (H Full Model)

- The models are simulated for one cycle of a sinusoidal transport current with an amplitude of 11 A, and a frequency of 50 Hz.







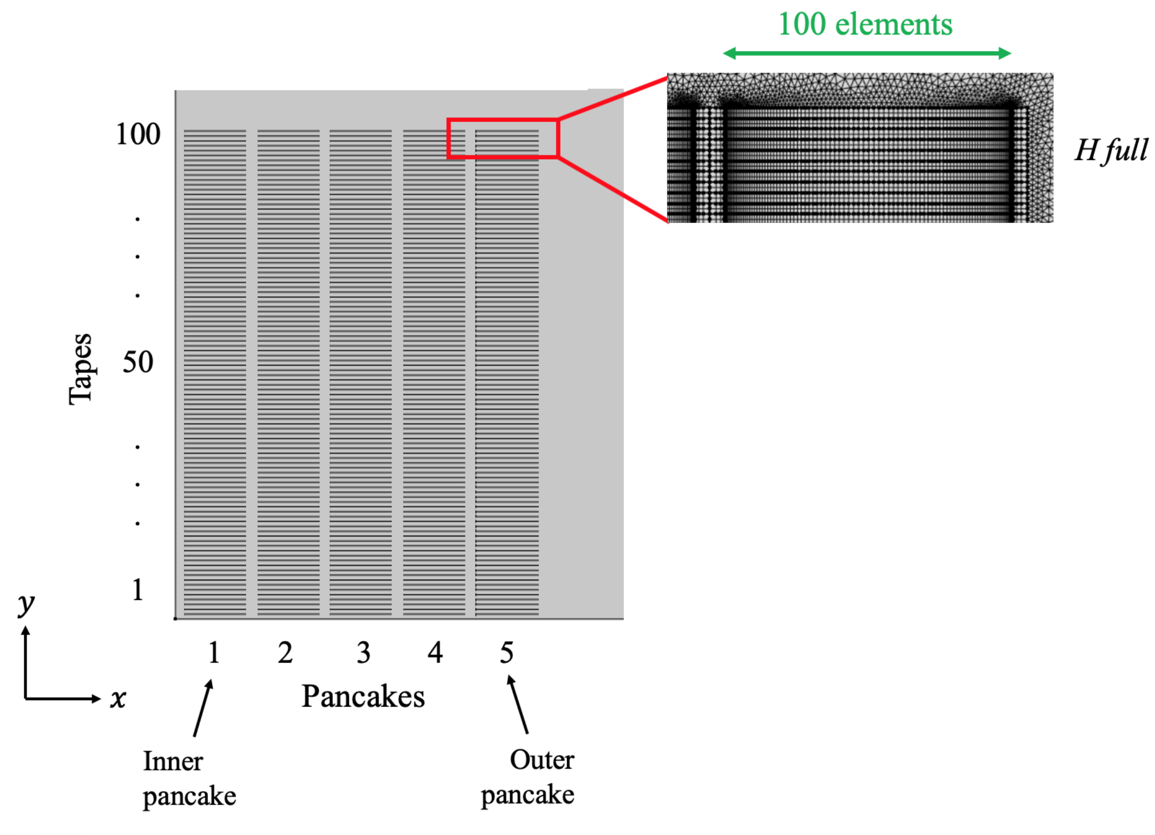


Average hysteresis losses $Q_{av} = 127.24$ W/m

Computation time $ct = 31$ h 32 min

Limitations of the *H* Full Model

- The *H* formulation has become the de facto standard within the community.
- The application of the FEM and the *H* formulation to large-scale systems is impaired by excessive computation times and memory requirements.



Homogenization and **multi-scale** strategies have been proposed to increase the computational efficiency.

Homogenization and H Formulation

Homogenization and *H* Formulation

- The homogenizations tranforms a stack of HTS tapes into an anisotropic bulk, such that the geometrical layout of the internal alternating structures is “washed” out while keeping the overall electromagnetic behavior *[Zermeño et al., 2012].*

JOURNAL OF APPLIED PHYSICS **114**, 173901 (2013)

CrossMark

Calculation of alternating current losses in stacks and coils made of second generation high temperature superconducting tapes for large scale applications

Victor M. R. Zermeno,[1,2,a)] Asger B. Abrahamsen,[3] Nenad Mijatovic,[4] Bogi B. Jensen,[4] and Mads P. Sørensen[2]
[1]*Karlsruhe Institute of Technology, P.O. Box 3640, 76021 Karlsruhe, Germany*
[2]*Department of Mathematics, Technical University of Denmark, Anker Engelunds Vej 1, 2800 Kgs. Lyngby, Denmark*
[3]*Department of Wind Energy, Technical University of Denmark, Frederiksborgvej 399, 4000 Roskilde, Denmark*
[4]*Department of Electrical Engineering, Technical University of Denmark, Anker Engelunds Vej 1, 2800 Kgs. Lyngby, Denmark*

(Received 12 August 2013; accepted 9 October 2013; published online 1 November 2013)

A homogenization method to model a stack of second generation High Temperature Superconducting tapes under AC applied transport current or magnetic field has been obtained. The idea is to find an anisotropic bulk equivalent for the stack such that the geometrical layout of the internal alternating structures of insulating, metallic, superconducting, and substrate layers is “washed” out while keeping the overall electromagnetic behavior of the original stack. We disregard assumptions upon the shape of the critical region and use a power law E–J relationship allowing for overcritical current densities to be considered. The method presented here allows for a computational speedup factor of up to 2 orders of magnitude when compared to full 2-D simulations taking into account the actual dimensions of the stacks without compromising accuracy. © *2013 AIP Publishing LLC*.
[http://dx.doi.org/10.1063/1.4827375]

I. INTRODUCTION

Second generation (2G) High Temperature Superconductor (HTS) coated conductors have already come to play an important role in a large number of power applications. Nowadays, superconducting cables such as the increasingly popular Roebel are being used for their high current capacity.[1,2] In the same manner, motors, generators, transformers, and large magnets are designed and/or built taking advantage of the high magnetic field achieved by superconducting coils or windings in compact designs. Although some of these devices are designed so that their superconducting elements do not experience AC electromagnetic fields, hysteretic losses are expected during start up, turn off, and other transient operations. Furthermore, transformers, asynchronous rotating machinery, and cables carrying AC currents are inherently burdened by hysteretic losses. Understanding and calcu-

The large aspect ratio of the thin films in 2G HTS coated conductors shows the multiscale nature of the layout: thickness and width are in different spatial scales. This later problem was already addressed in an earlier work of ours[3] where structured meshes were used to achieve a computational speedup of 2–3 orders of magnitude. Modeling and simulation of large stacks of 2G HTS coated conductors under AC conditions has already been the subject of study by means of: integral equations using a thin conductor approximation for the cases of infinite stacks, periodic arrays, or a couple of conductors;[4] direct integration also using a thin conductor approximation for the cases of infinite bifilar stacks;[5] quasi-variational inequalities[6] detailing the actual layout of just a few conductors; partial differential equations describing stacks of up to 100 conductors;[3,7] and anisotropic homogenous-medium approximations[8–10] used for arbitrarily large stacks.

Homogenization and *H* Formulation

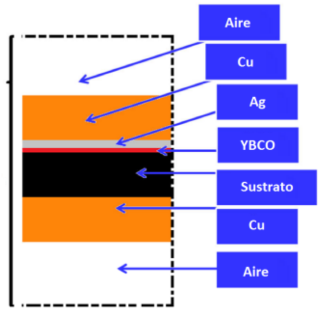
- The homogenizations tranforms a stack of HTS tapes into an anisotropic bulk, such that the geometrical layout of the internal alternating structures is “washed” out while keeping the overall electromagnetic behavior *[Zermeño et al., 2012].*
- A new equivalent critical current density is defined

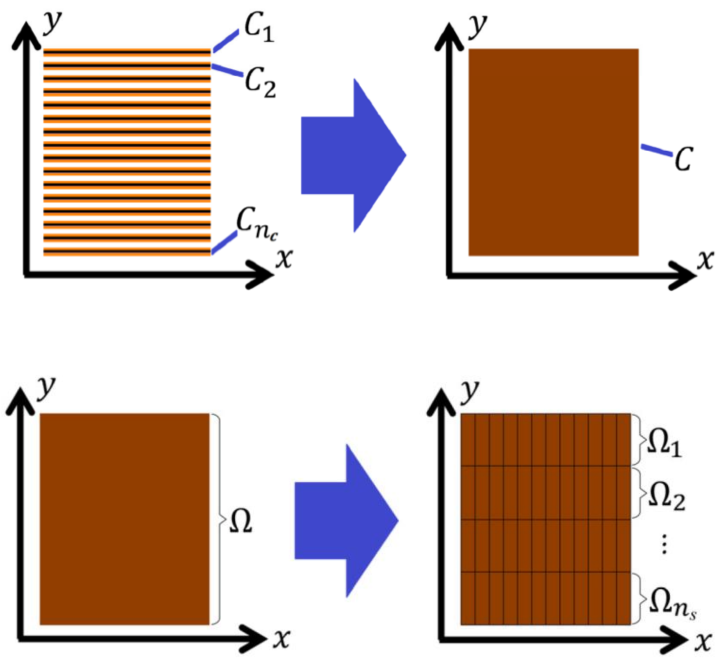
$$J_{c,eq}(B,\theta) = f_{HTS} \cdot J_c(B,\theta)$$

where f_{HTS} is the volume fraction of the superconducting material.

- The homogenized stack is divided in subsets, and a new transport current is impressed in each subset, depending on the turns it represents

$$\widehat{I_k}(t) = \int_{\Omega_k} J(x,y,t)dxdy$$





[Zermeño et al., 2012].

Definitions

The average hysteresis losses are obtained using data of the second half of the cycle, as follows,

$$Q_{av} = \frac{2}{P}\int_{P/2}^{P}\int_{\Omega_{sc}} \mathbf{E}\cdot\mathbf{J}\, dS\, dt,$$

where P is the period of the sinusoidal cycle, and Ω_{sc} are the superconducting subdomains.

The average losses relative error, expressed in percent, is defined as,

$$er_Q = \frac{(Q_{M\_av} - Q_{R\_av})}{Q_{R\_av}} \times 100\ \%,$$

where Q_{R_av} and Q_{M_av} are the average losses computed with the reference and with the model that is being compared, respectively.

The J distributions are multivariable functions. The coefficient of determination is defined as,

$$R^2 = 1 - \frac{\sum_{i=1}^{m}(J_R - J_M)^2}{\sum_{i=1}^{m}(J_R - \bar{J}_M)^2},$$

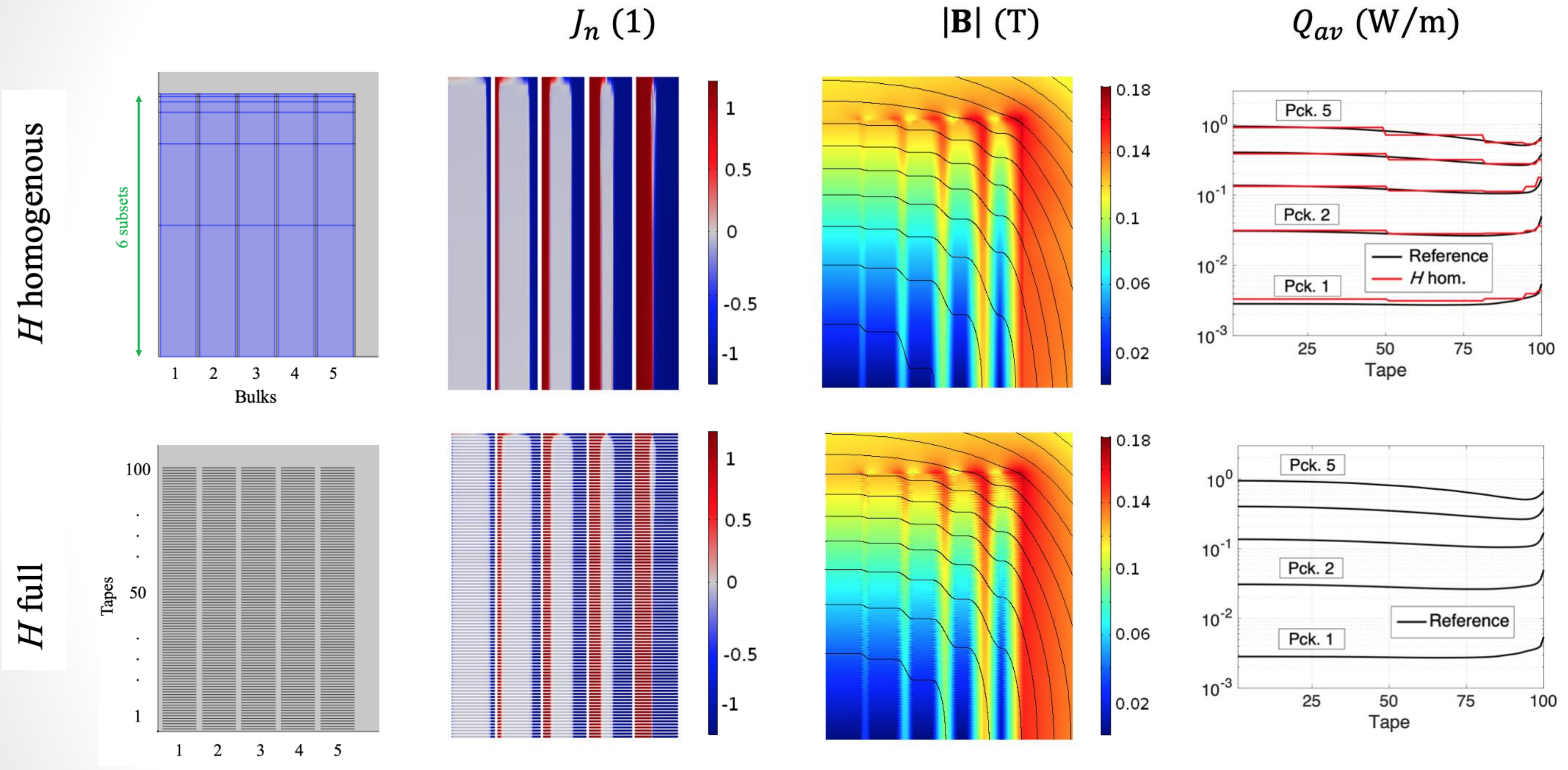
where J_R and J_M are vectors built by concatenating the J distribution of all the tapes for all the time steps, computed with the reference model and with the tested model, respectively. $\bar{J}_R$ is the mean value of J_R. It must be remembered that $R^2 = 1$ means a perfect matching between J_R and J_M.

The normalized computation time is defined as,

$$\bar{ct} = \frac{ct_R}{ct_M},$$

where ct_R and ct_M are the computation time required by the reference model and by the tested model, respectively.

Case Study *H* Homogeneous Model



J coef. of det. $R^2 = 0.9221$ Losses error $er_Q = 1.28\ \%$ Normalized comp. time $\bar{ct} = 1.94\ \%$

Multi-scaling
and
H Formulation

Multi-scaling and *H* Formulation

- The multi-scale model is composed by the coil submodel and the single-tape submodel [*Queval et al., 2016*].

IOP Publishing Superconductor Science and Technology

Supercond. Sci. Technol. **29** (2016) 024007 (10pp) doi:10.1088/0953-2048/29/2/024007

Numerical models for ac loss calculation in large-scale applications of HTS coated conductors

Loïc Quéval[1,3], **Víctor M R Zermeño**[2] **and Francesco Grilli**[2]

[1] University of Applied Sciences Düsseldorf, Josef-Gockeln Strasse 9, 40474 Düsseldorf, Germany
[2] Karlsruhe Institute of Technology, Hermann-von-Helmholtz Platz 1, 76344 Eggenstein-Leopoldshafen, Germany

E-mail: loic.queval@gmail.com, victor.zermeno@kit.edu and francesco.grilli@kit.edu

Received 1 June 2015, revised 13 October 2015
Accepted for publication 15 October 2015
Published 12 January 2016

CrossMark

Abstract

Numerical models are powerful tools to predict the electromagnetic behavior of superconductors. In recent years, a variety of models have been successfully developed to simulate high-temperature-superconducting (HTS) coated conductor tapes. While the models work well for the simulation of individual tapes or relatively small assemblies, their direct applicability to devices involving hundreds or thousands of tapes, e.g., coils used in electrical machines, is questionable. Indeed, the simulation time and memory requirement can quickly become prohibitive. In this paper, we develop and compare two different models for simulating realistic HTS devices composed of a large number of tapes: (1) the homogenized model simulates the coil using an equivalent anisotropic homogeneous bulk with specifically developed current constraints to account for the fact that each turn carries the same current; (2) the multi-scale model parallelizes and reduces the computational problem by simulating only several individual tapes at significant positions of the coil's cross-section using appropriate boundary conditions to account for the field generated by the neighboring turns. Both methods are used to simulate a coil made of 2000 tapes, and compared against the widely used H-formulation finite-element model that includes all the tapes. Both approaches allow faster simulations of large number of HTS tapes by 1–3 orders of magnitudes, while maintaining good accuracy of the results. Both models can therefore be used to design and optimize large-scale HTS devices. This study provides key advancement with respect to previous versions of both models. The homogenized model is extended from simple stacks to large arrays of tapes. For the multi-scale model, the importance of the choice of the current distribution used to generate the background field is underlined; the error in ac loss estimation resulting from the most obvious choice of starting from a uniform current distribution is revealed.

Keywords: ac losses, numerical models, superconducting coils

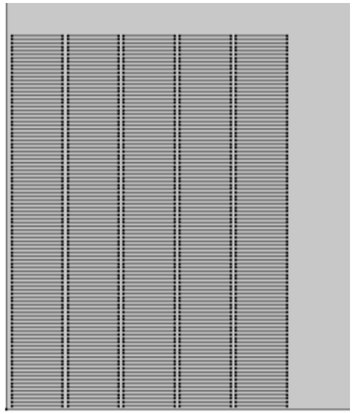
Multi-scaling and *H* Formulation

- The multi-scale model is composed by the coil submodel and the single-tape submodel [*Queval et al., 2016*].
- The H field is estimated with the coil submodel.
- The H field along the boundary of the analyzed tapes is exported to the single tape submodel as a time-dependent Dirichlet boundary condition.

Coil Submodel

- 2D planar model.
- Includes all the tapes.
- A-formulation magnetostatic.

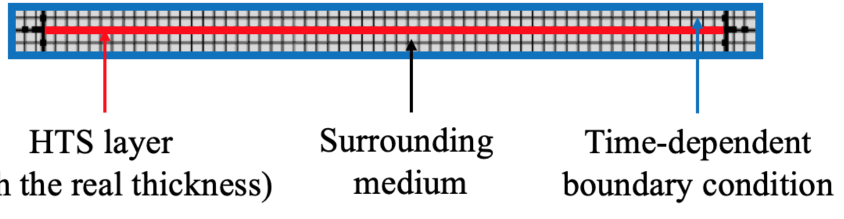
$$\nabla \times \nabla \times \mathbf{A} = \mu \mathbf{J}$$



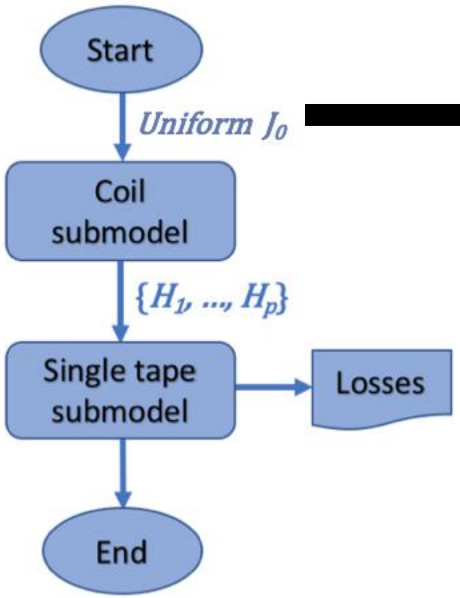
Single Tape Submodel

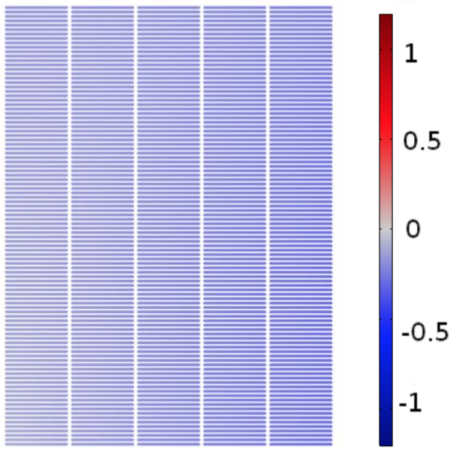
- 2D planar model.
- Includes only one tape.
- *H* formulation dynamic model.

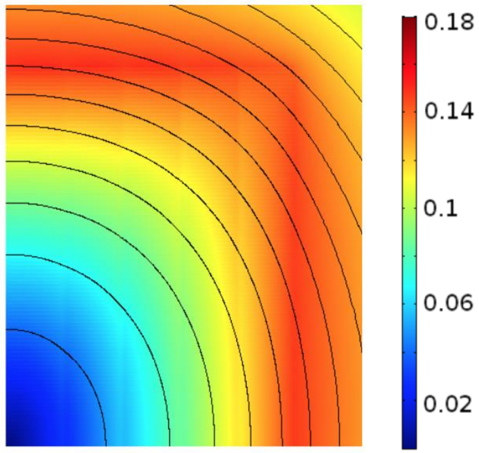
$$\nabla \times \rho \nabla \times \mathbf{H} = -\mu \frac{\partial(\mathbf{H})}{\partial t}$$



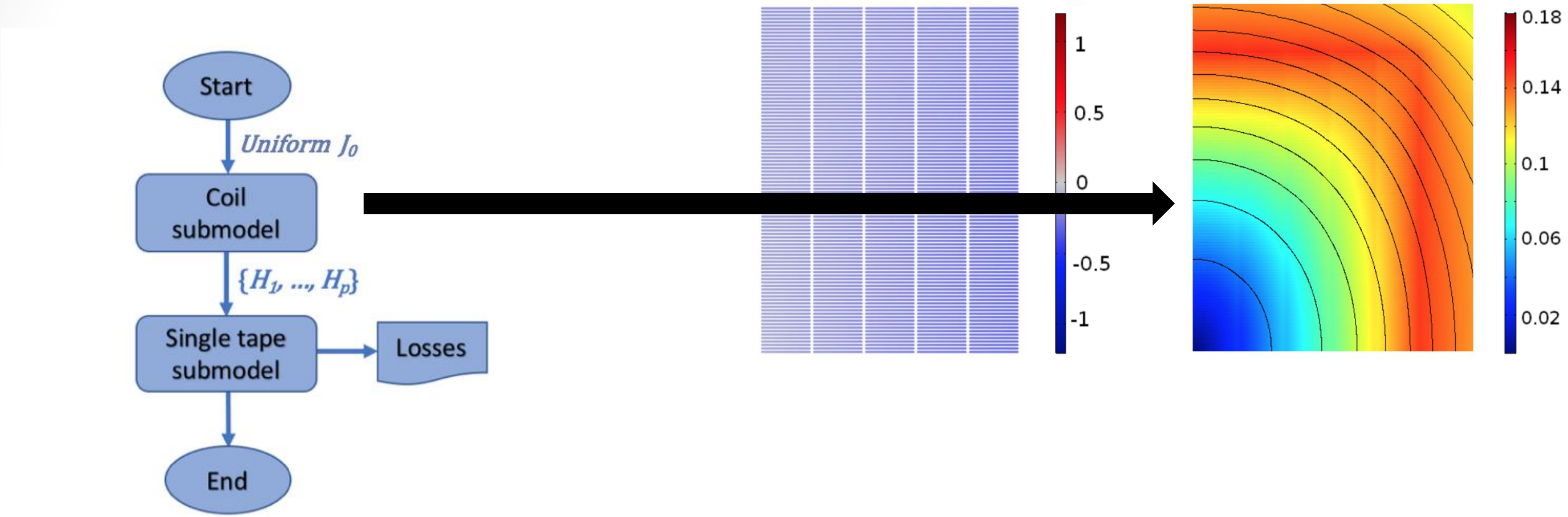
Multi-scaling and *H* Formulation



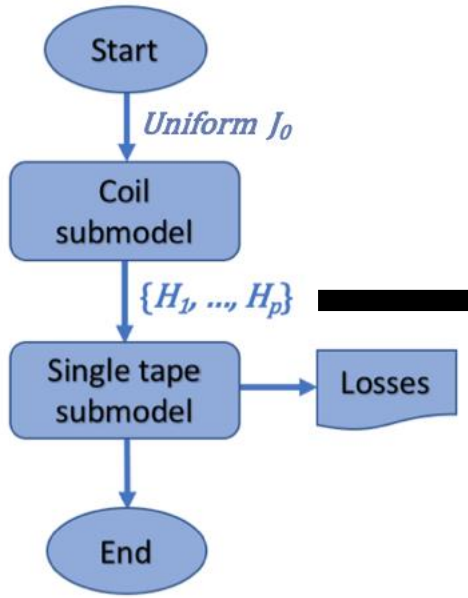


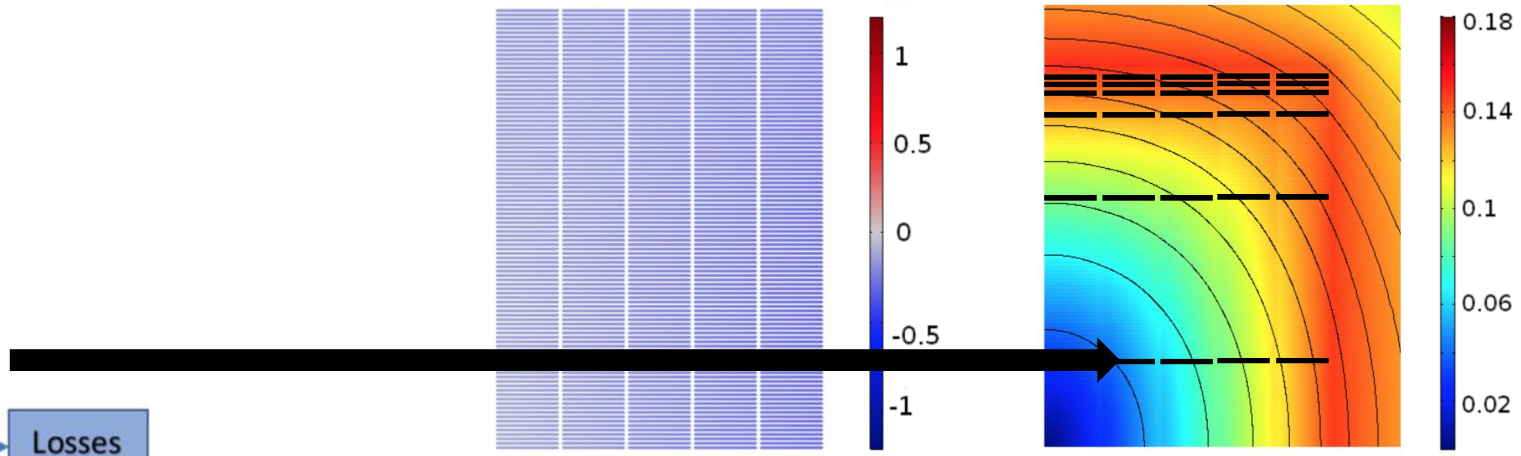


Multi-scaling and *H* Formulation

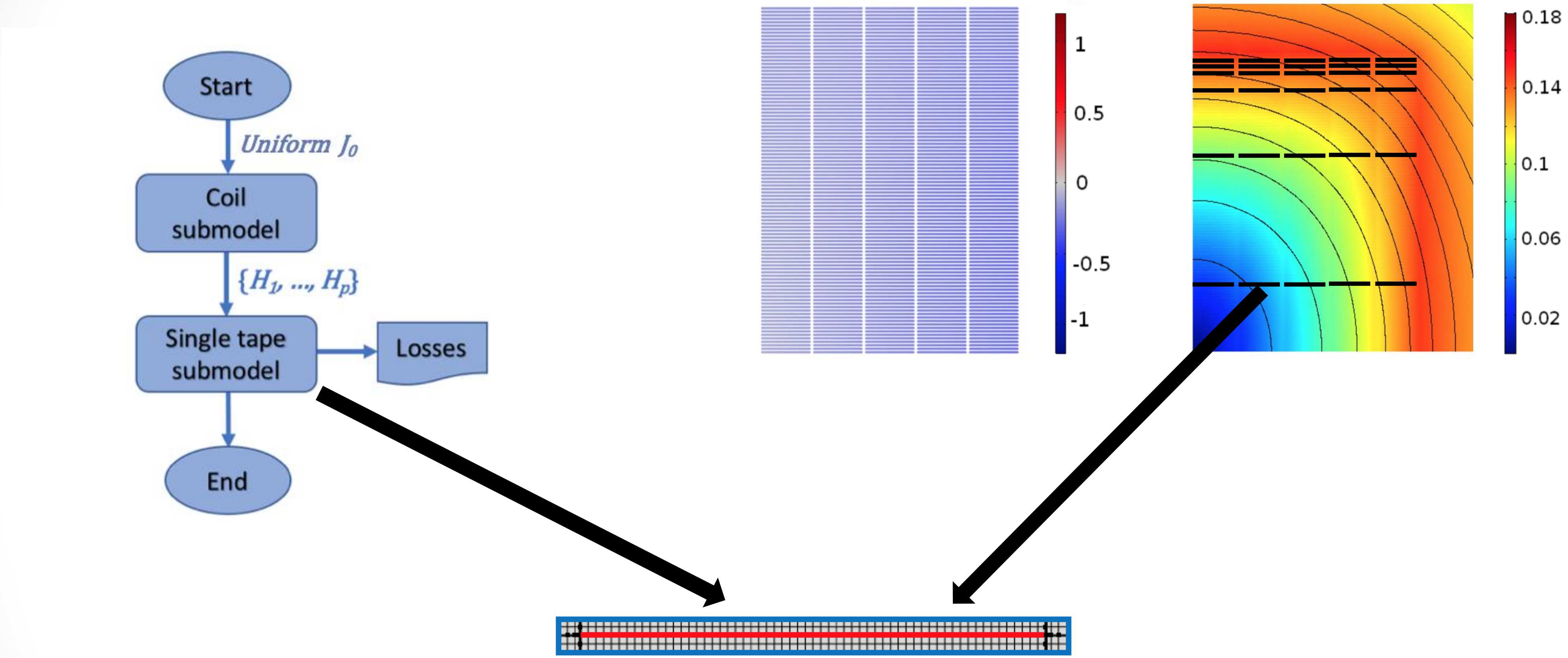


Multi-scaling and *H* Formulation

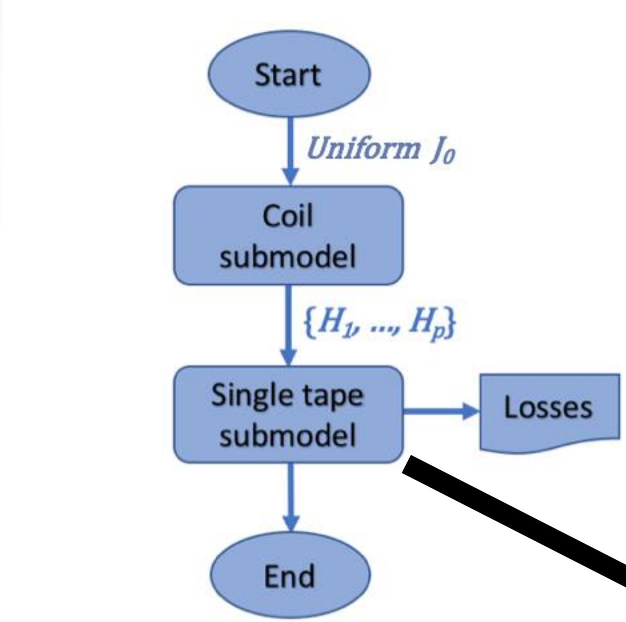


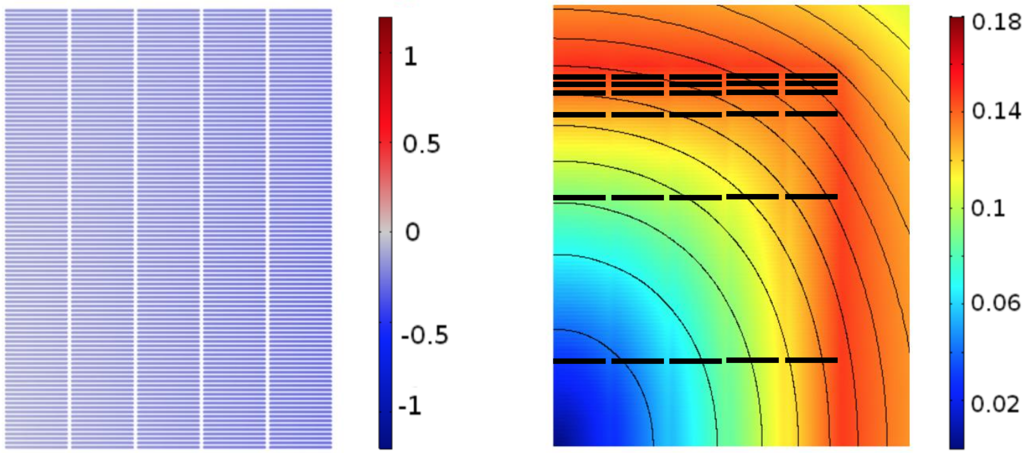


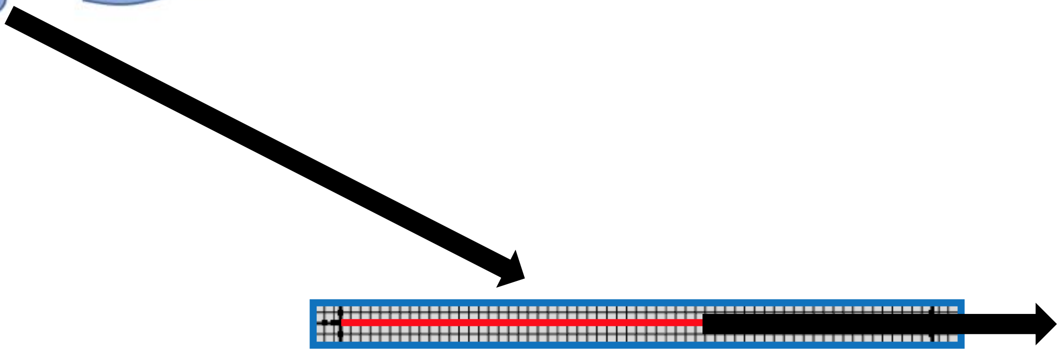
Multi-scaling and *H* Formulation

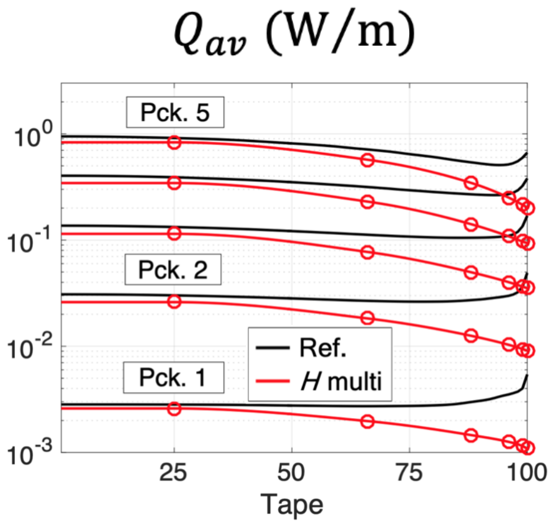


Multi-scaling and *H* Formulation

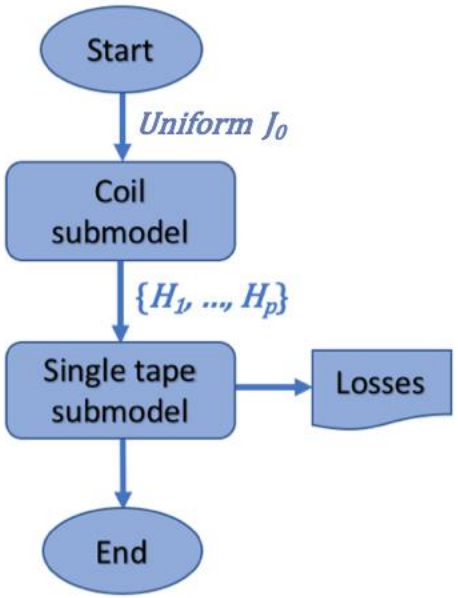


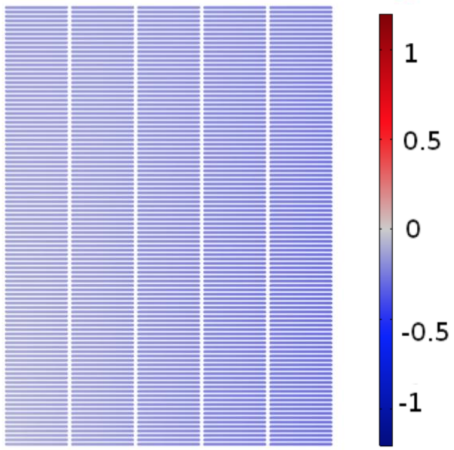


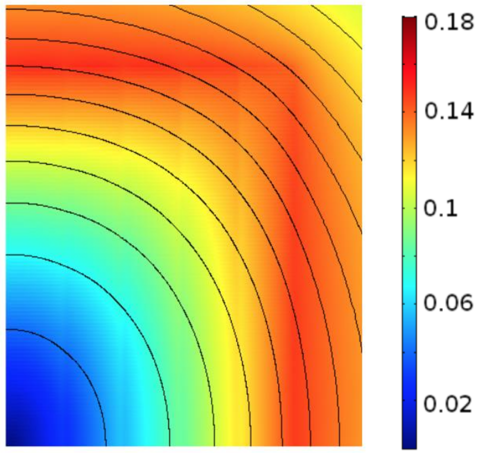




Multi-scaling and *H* Formulation





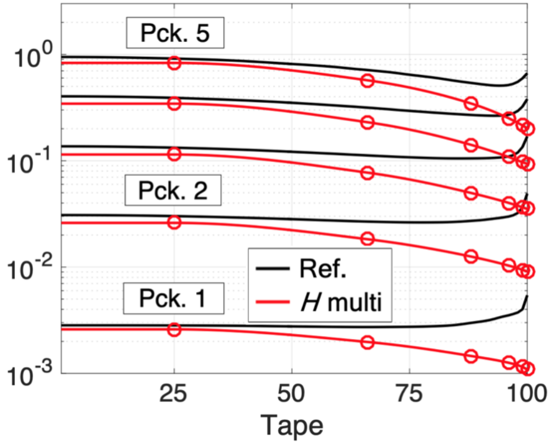


$$er_Q = -21.7\ \%$$

$$R^2 = 0.0302$$

$$\overline{ct} = 1.45\ \%$$

Q_{av} (W/m)



Iterative Multi-scaling and *H* Formulation

- The iterative multi-scaling strategy the iterative implementation of the multi-scaling strategy [*Berrospe et al., 2018*].

IOP Publishing Superconductor Science and Technology

Supercond. Sci. Technol. **31** (2018) 095002 (13pp) https://doi.org/10.1088/1361-6668/aad224

Iterative multi-scale method for estimation of hysteresis losses and current density in large-scale HTS systems

Edgar Berrospe-Juarez[1], Víctor M R Zermeño[2], Frederic Trillaud[3] and Francesco Grilli[2]

[1] Postgraduate School of Engineering, National Autonomous University of Mexico, Mexico
[2] Karlsruhe Institute of Technology, Germany
[3] Institute of Engineering, National Autonomous University of Mexico, Mexico

E-mail: eberrospej@iingen.unam.mx

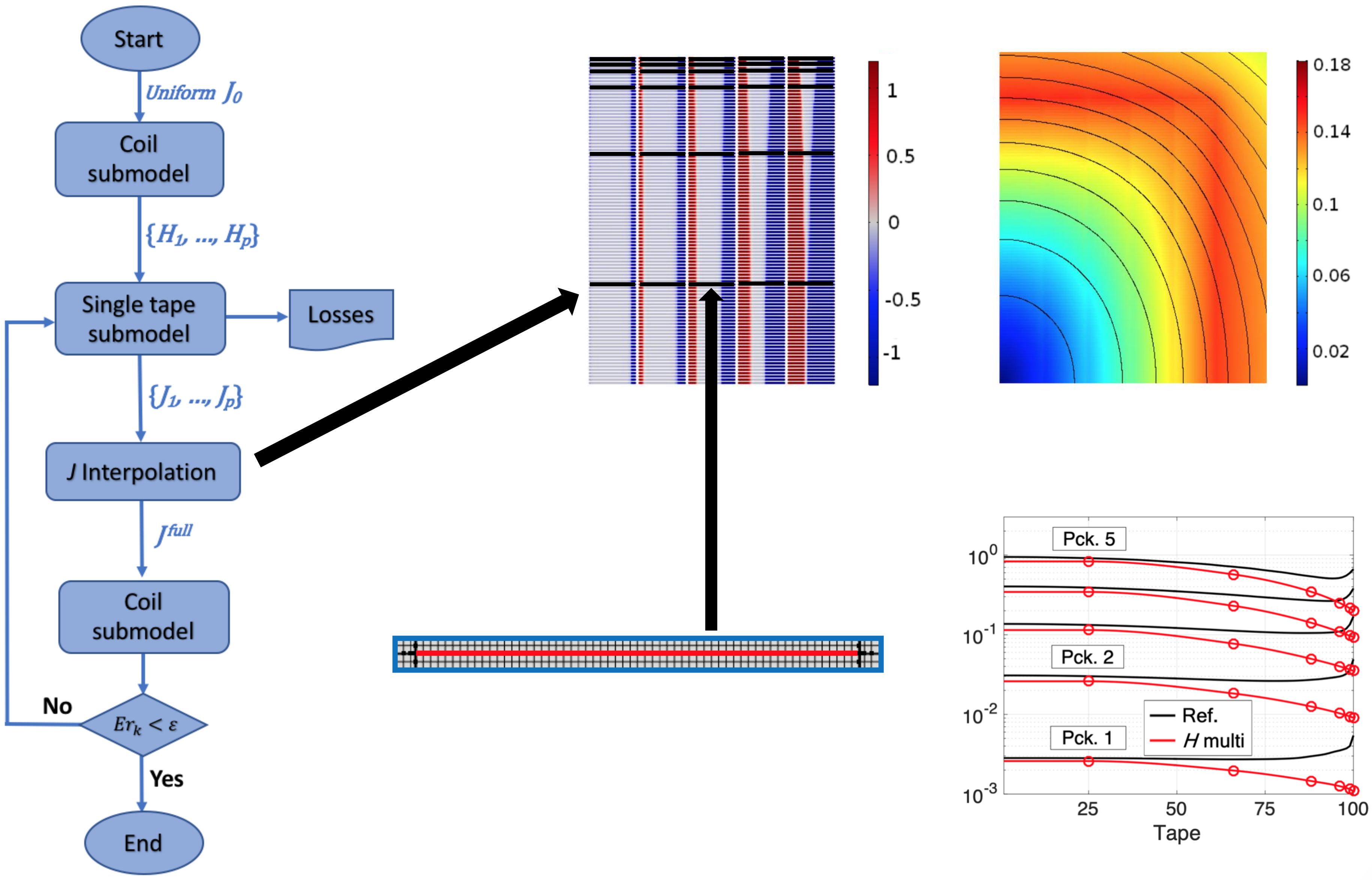
Received 24 April 2018, revised 8 June 2018
Accepted for publication 9 July 2018
Published 30 July 2018

CrossMark

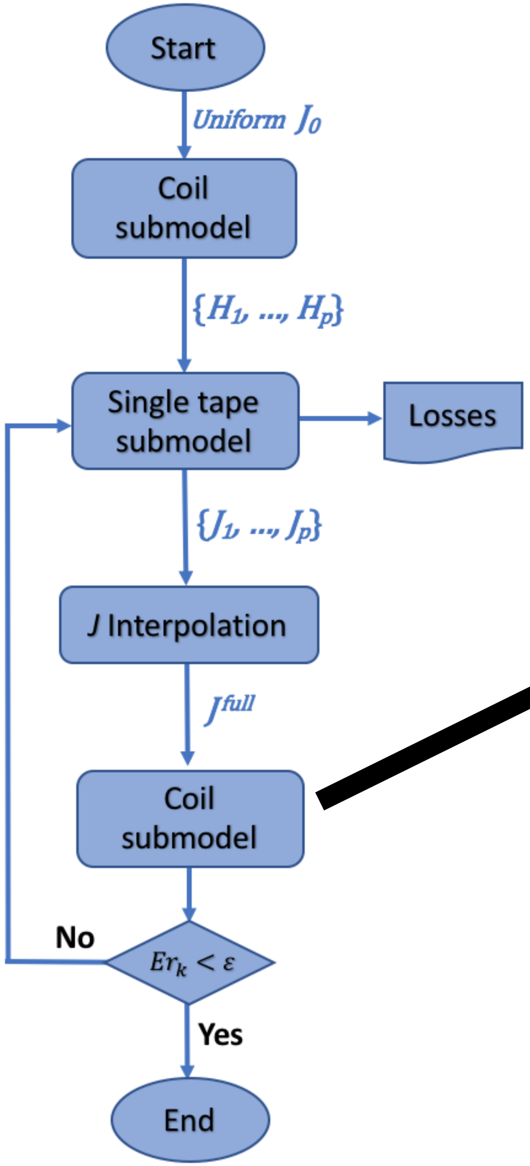
Abstract

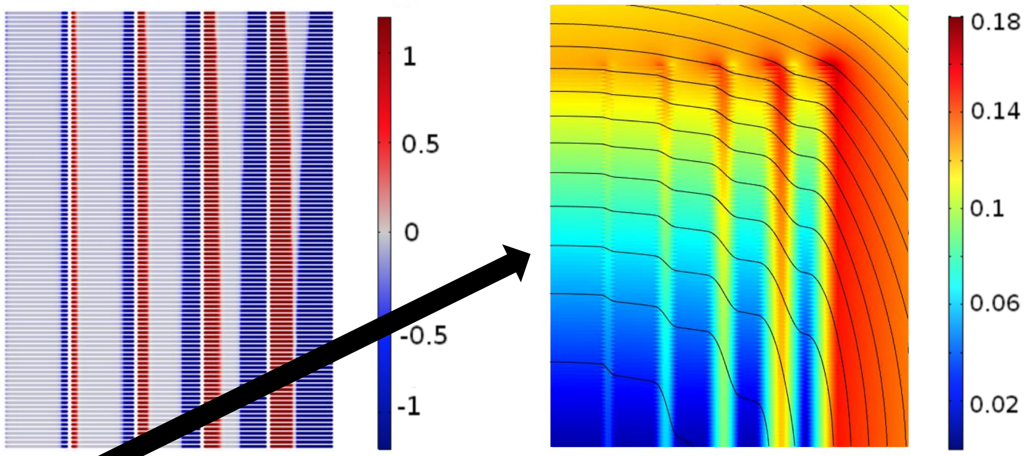
In recent years, commercial high-temperature superconductor (HTS) materials have gained increasing interest for their use in applications involving large-scale superconductor systems. These systems are typically made from hundreds to thousands of turns of conductors. These applications can range from power engineering devices like power transformers, motors and generators, to commercial and scientific magnets. The available analytical models are restricted to the analysis of individual tapes or relatively simple assemblies, therefore it is not possible to apply these models to the study of large-scale systems and other simulation tools are required. Due to the large number of turns, the simulations of a whole system can become prohibitive in terms of computing time and load. Therefore, an efficient strategy which does not compromise the accuracy of calculations is needed. Recently, a method, based on a multi-scale approach, showed that the computational load can be lowered by simulating, in detail, only several significant tapes from the system. The main limitation of this approach is the inaccuracy of the estimation of the background magnetic field, this means the field affecting the significant tapes produced by the rest of the tapes and by external sources. To address this issue, we consider the following two complementary strategies. The first strategy consists of the iterative implementation of the multi-scale method. The multi-scale method itself solves a dynamic problem, the iterative implementation proposed here is the iterative application of the multi-scale method, and a dynamic solution is obtained at each iteration. The

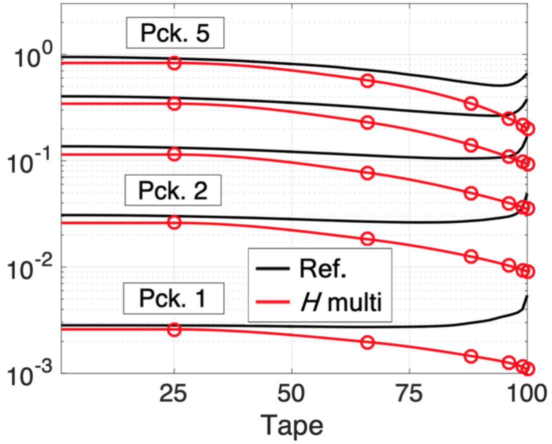
Iterative Multi-scaling and *H* Formulation



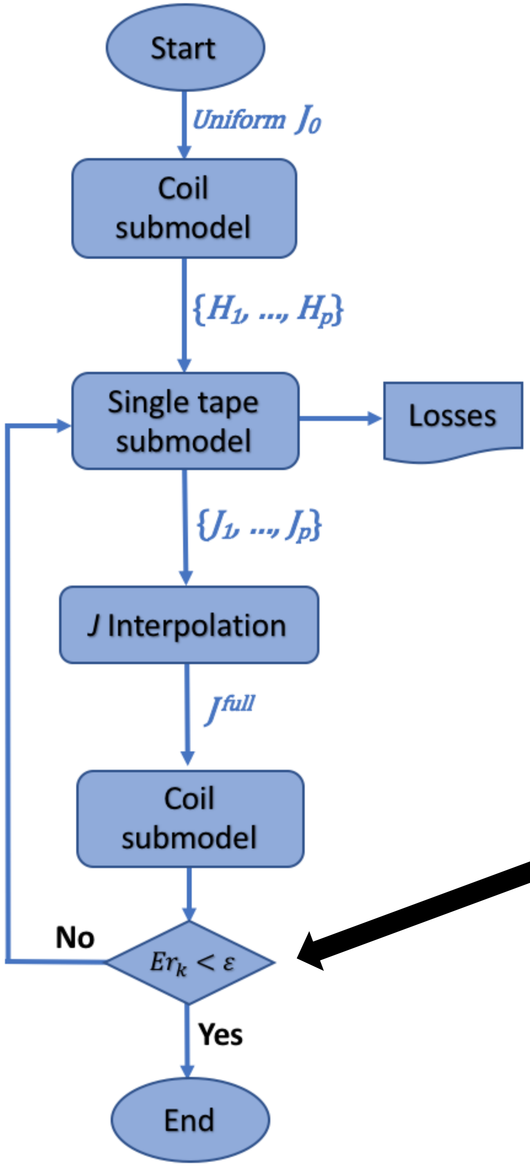
Iterative Multi-scaling and *H* Formulation

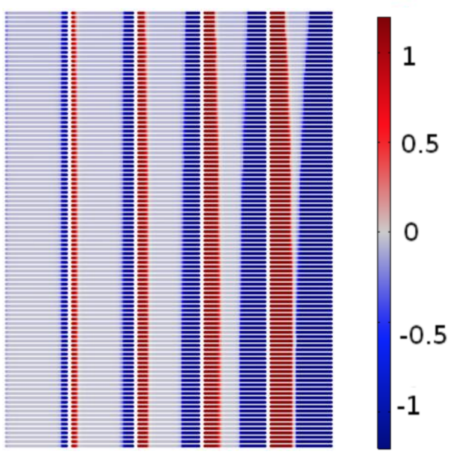


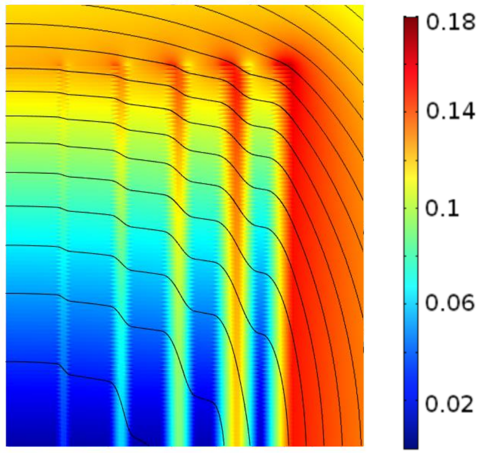




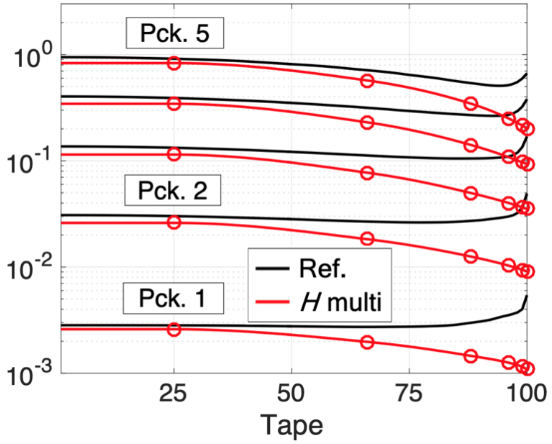
Iterative Multi-scaling and *H* Formulation



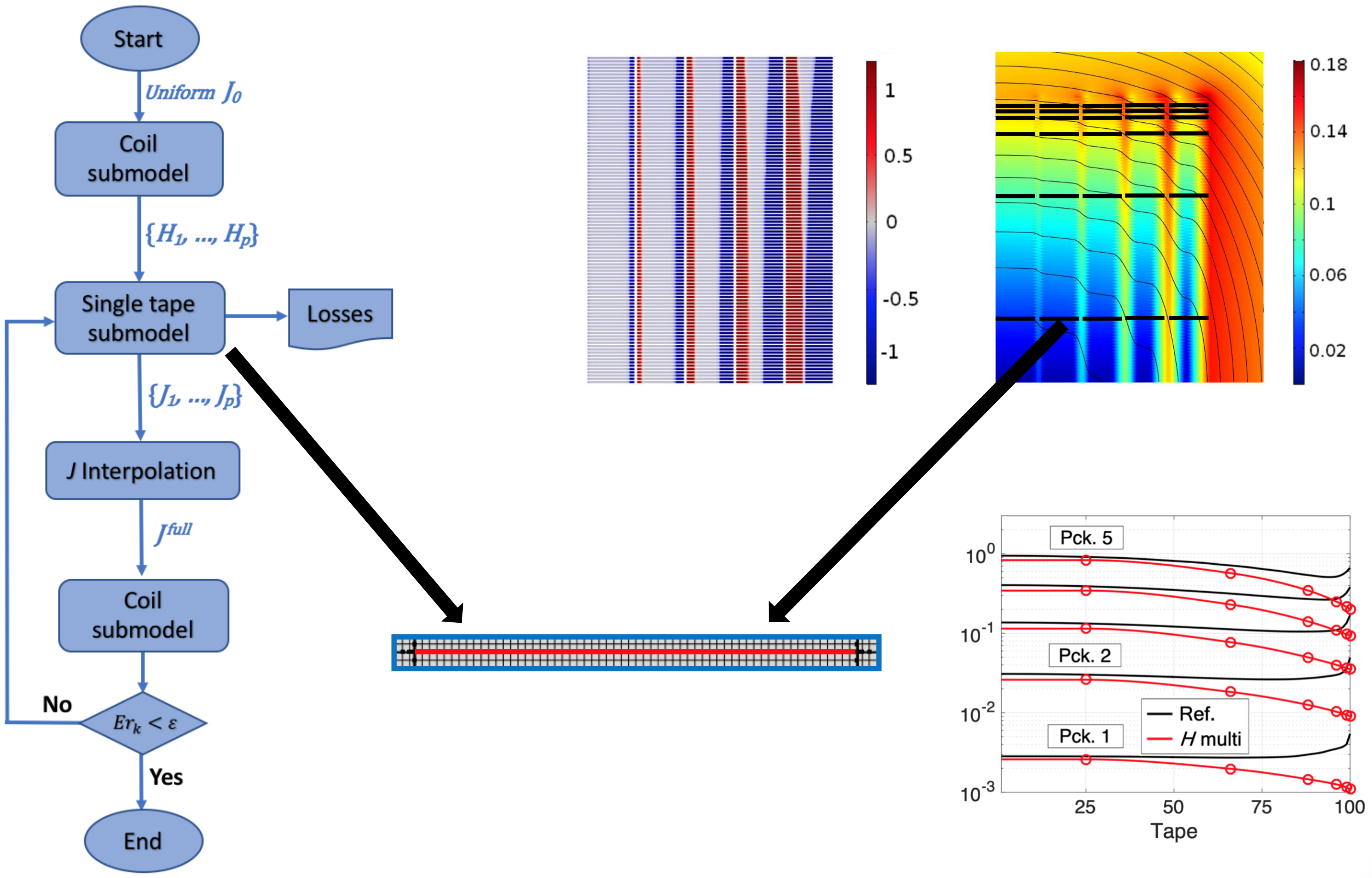




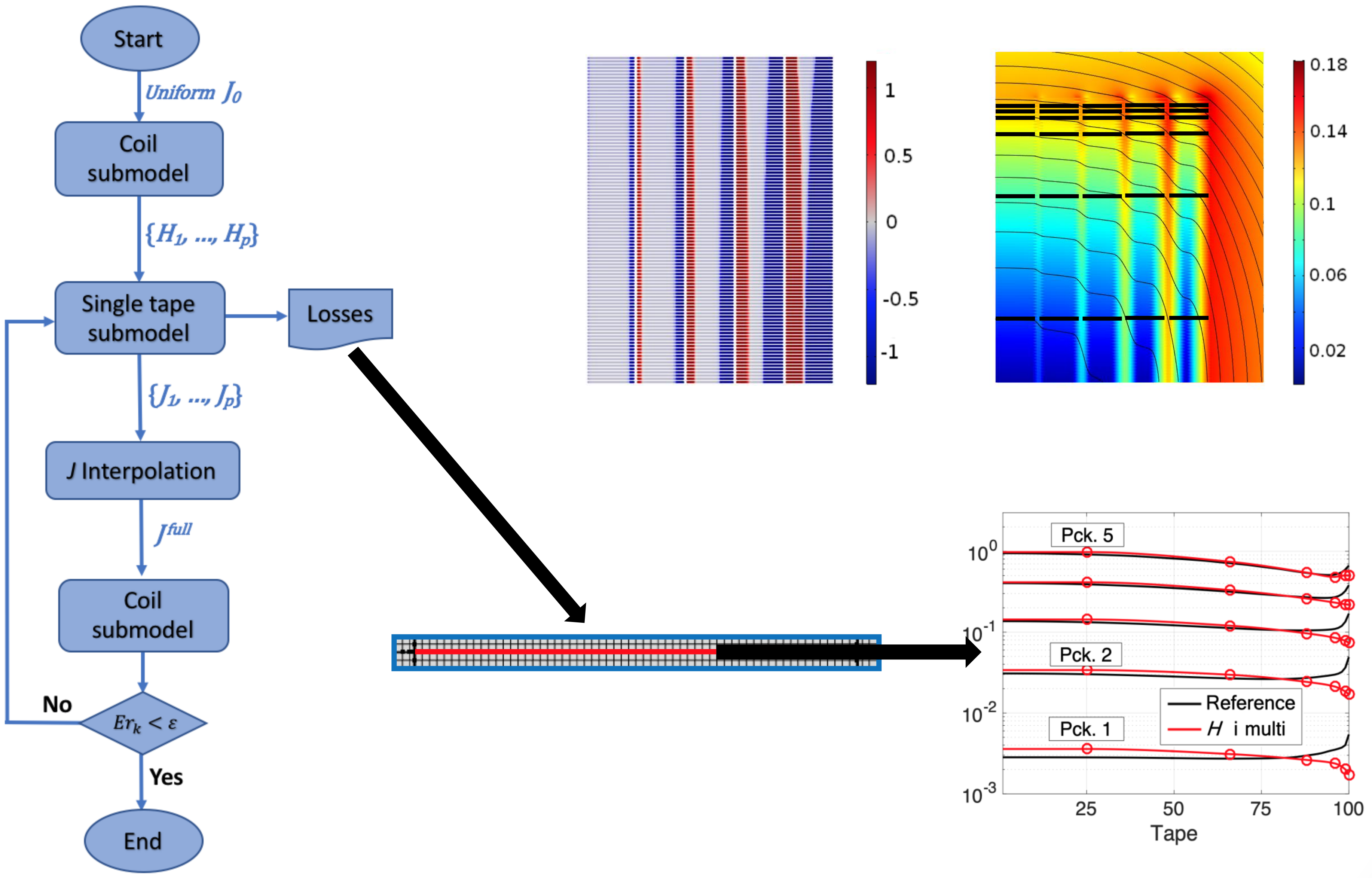
$$Er_k = \frac{\sqrt{\sum_{i=1}^{l}\left(J_i^{k-1} - J_i^{k}\right)^2}}{\sqrt{\sum_{i=1}^{l}\left(J_i^{k-1}\right)^2}}$$



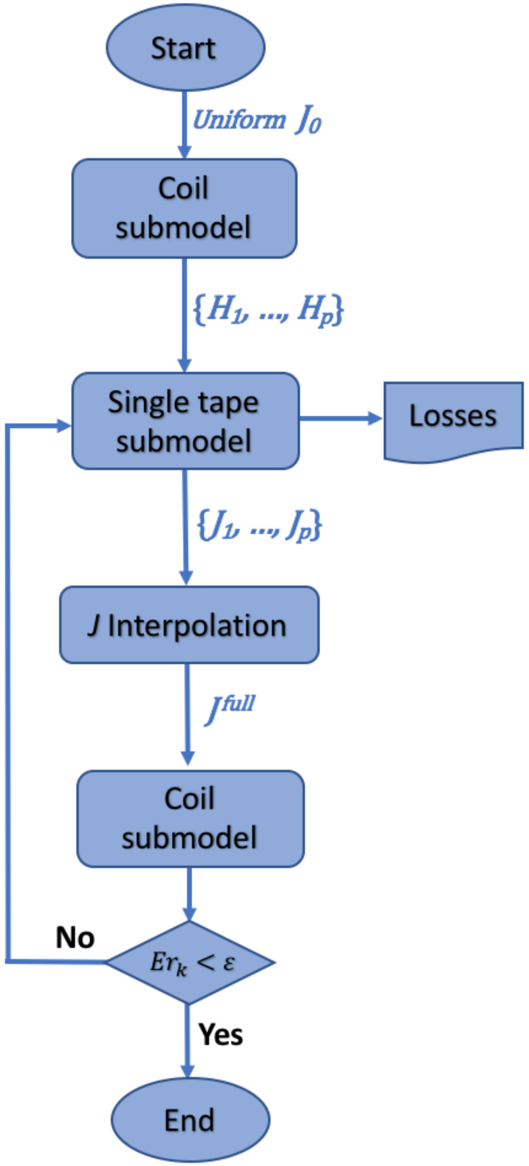
Iterative Multi-scaling and *H* Formulation

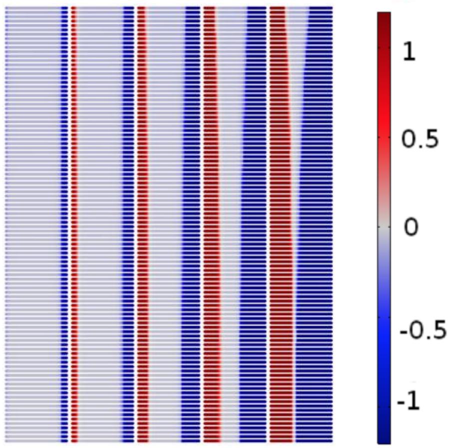


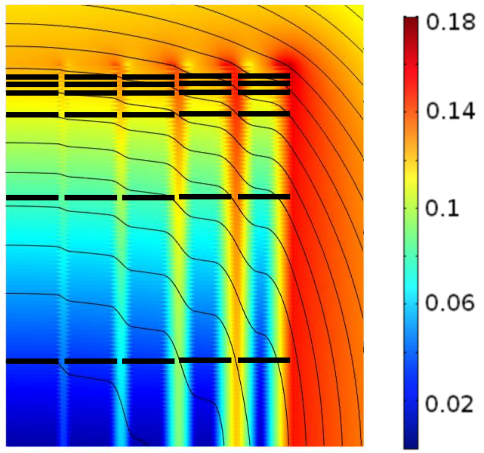
Iterative Multi-scaling and *H* Formulation



Iterative Multi-scaling and *H* Formulation



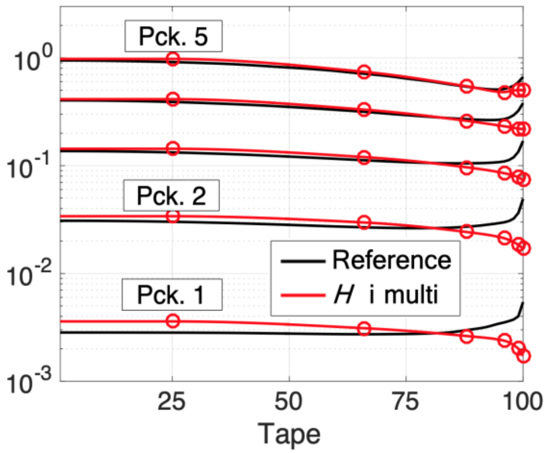




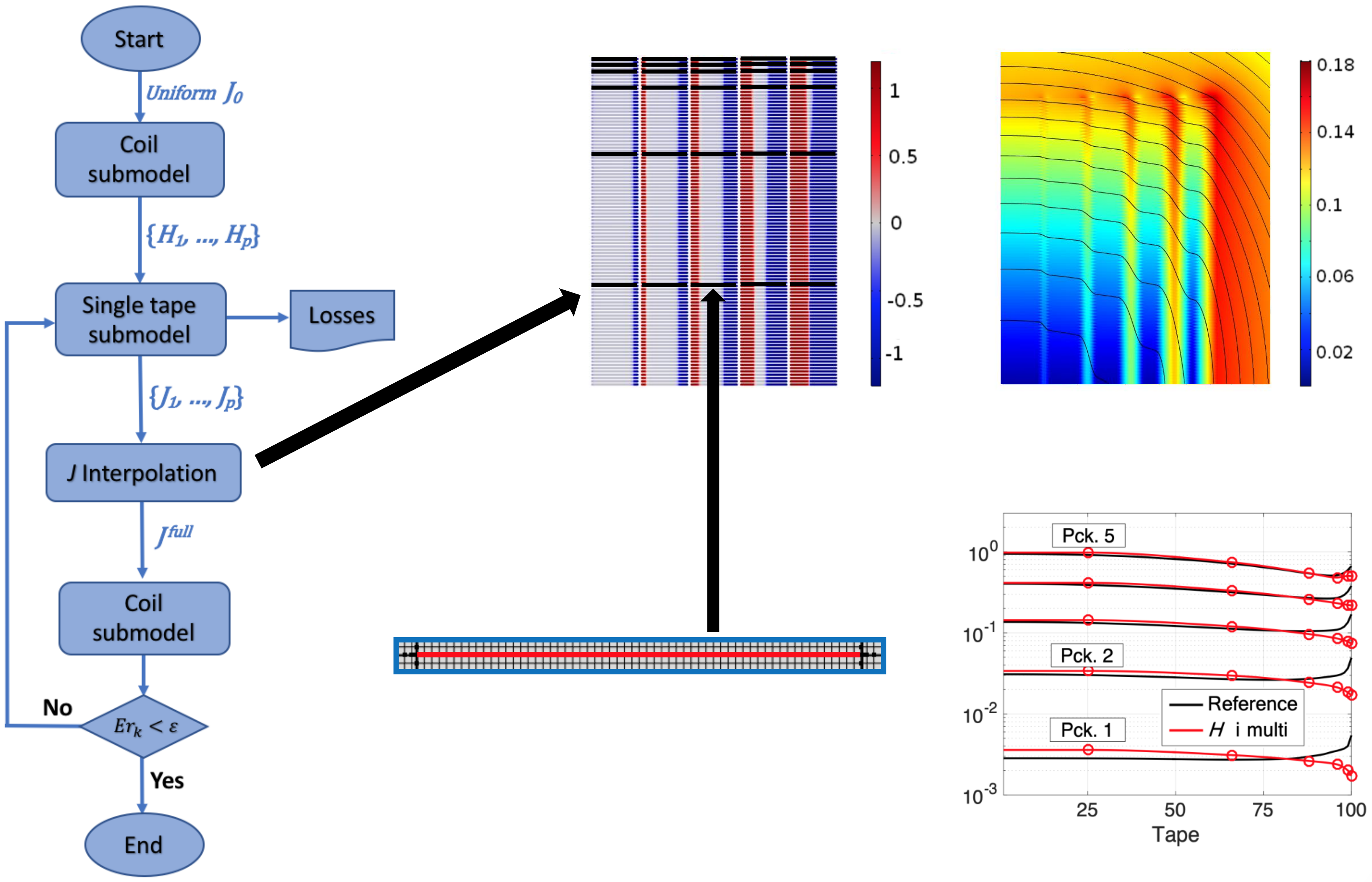
$$er_Q = 1.89\ \%$$

$$R^2 = 0.8306$$

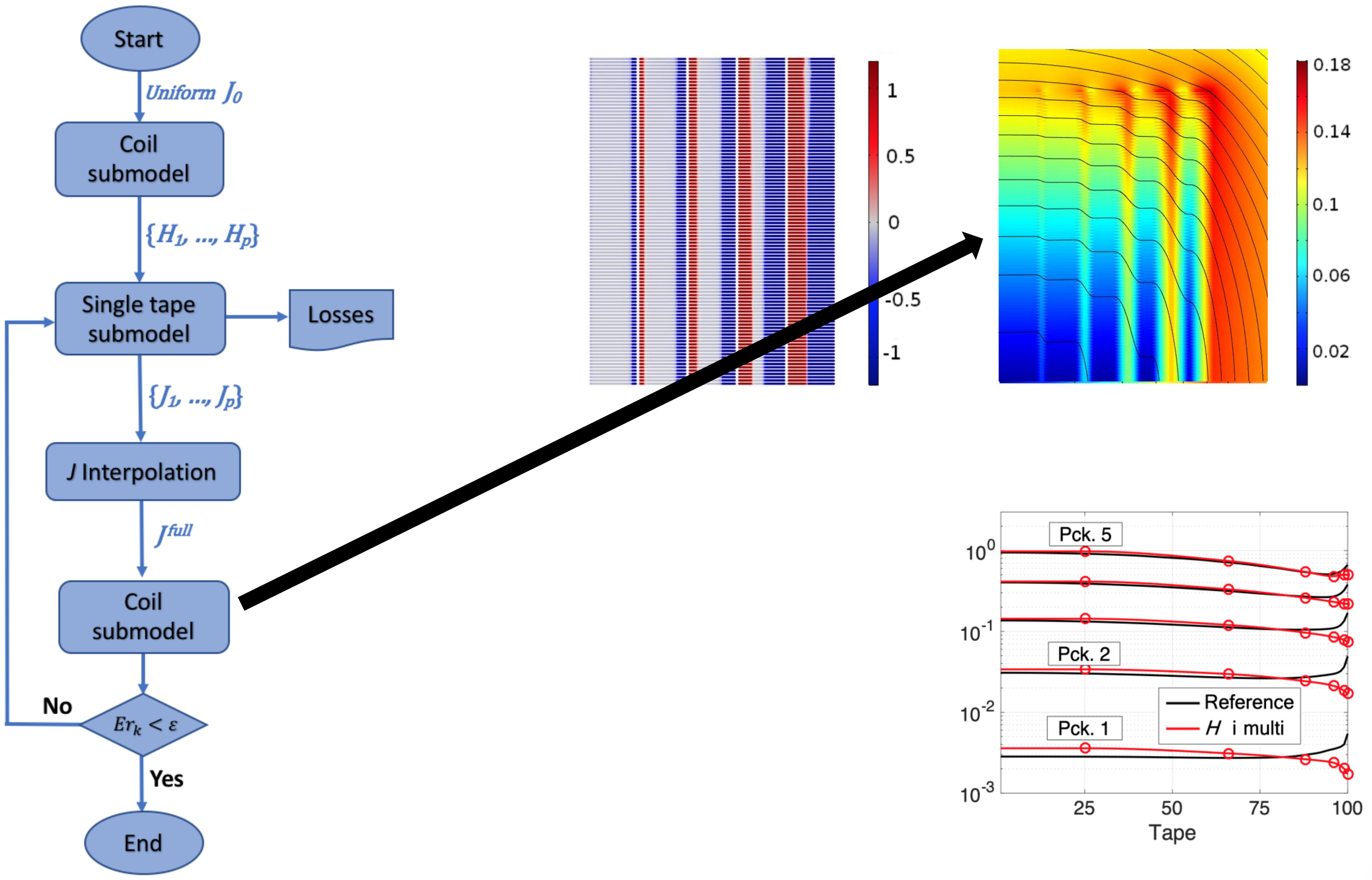
$$\overline{ct} = 2.85\ \%$$



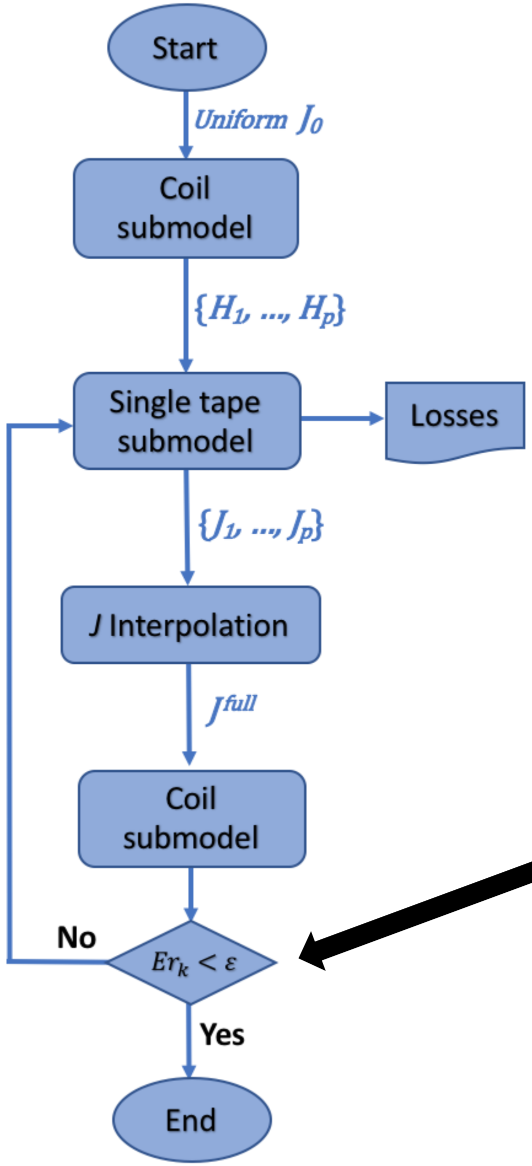
Iterative Multi-scaling and *H* Formulation

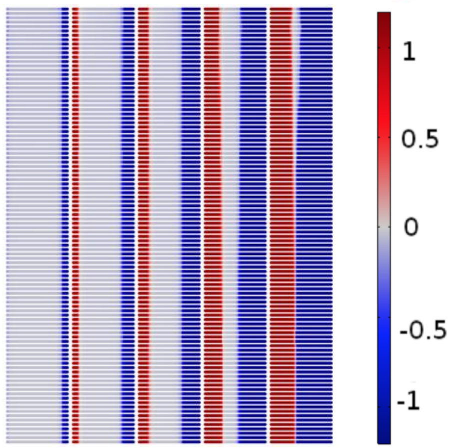


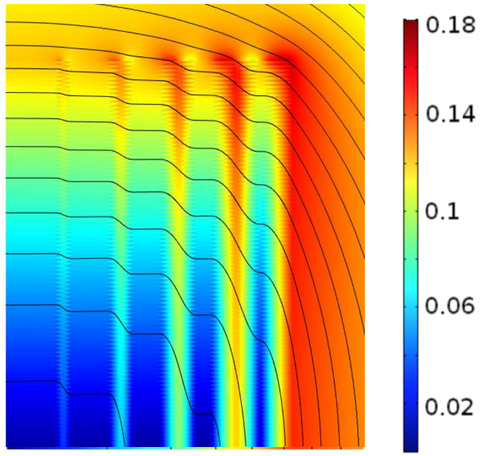
Iterative Multi-scaling and *H* Formulation



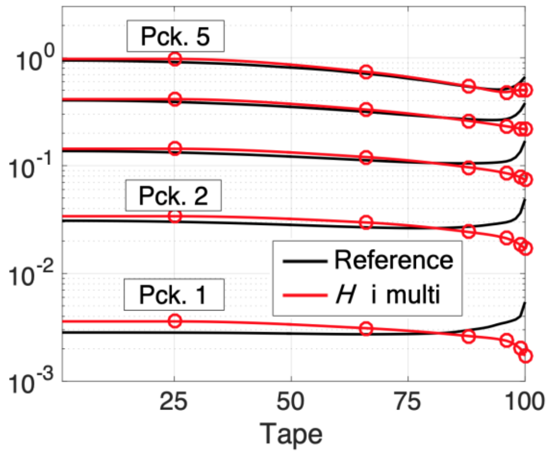
Iterative Multi-scaling and *H* Formulation



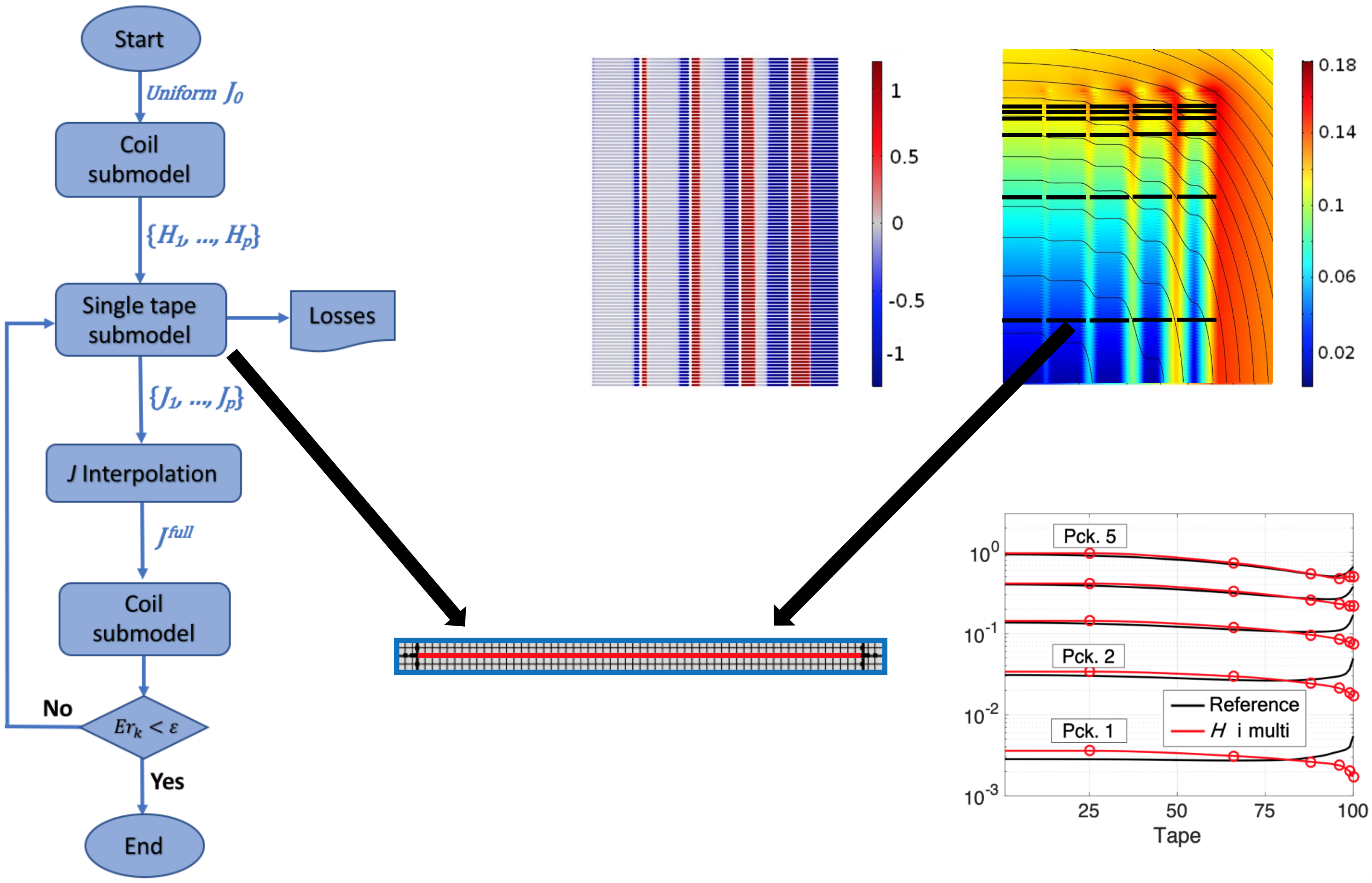




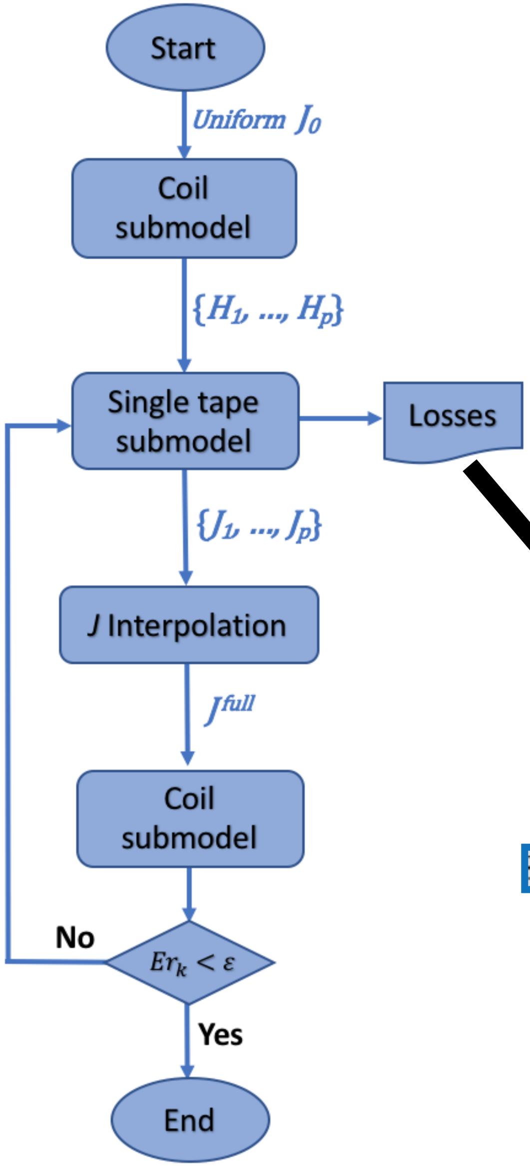
$$Er_k = \frac{\sqrt{\Sigma_{i=1}^{l}\left(J_i^{k-1} - J_i^{k}\right)^2}}{\sqrt{\Sigma_{i=1}^{l}\left(J_i^{k-1}\right)^2}}$$

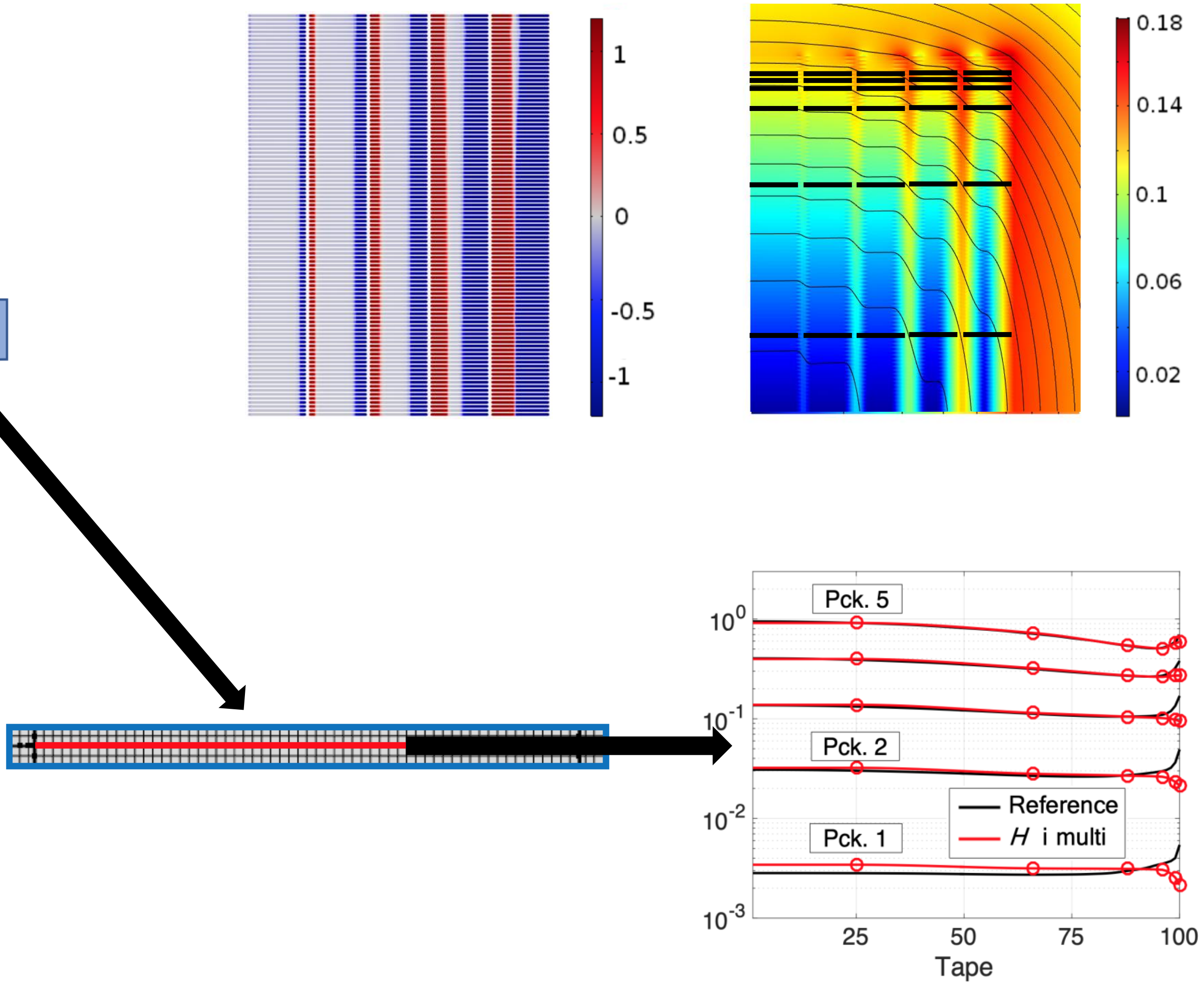


Iterative Multi-scaling and *H* Formulation

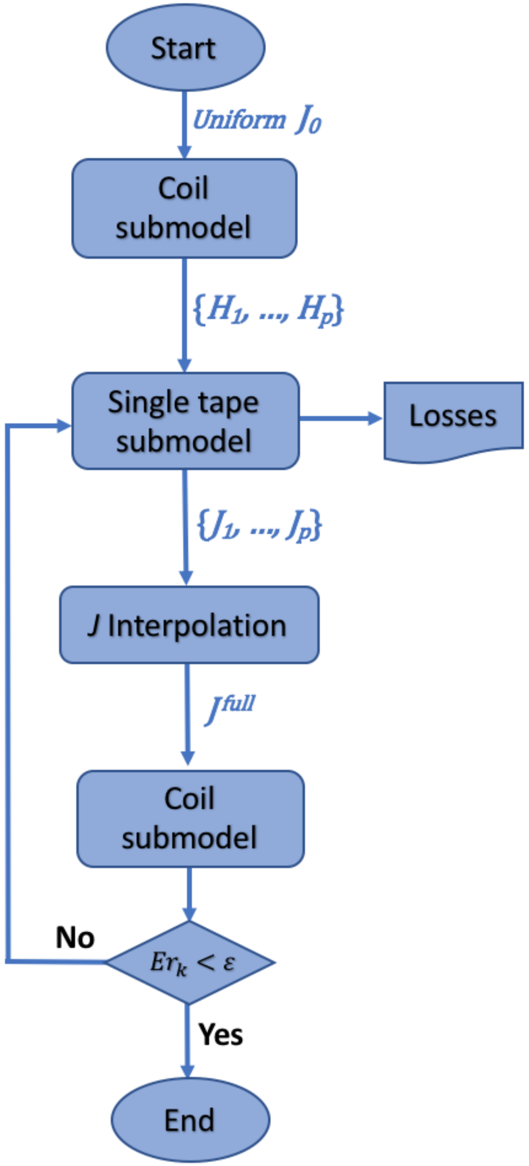


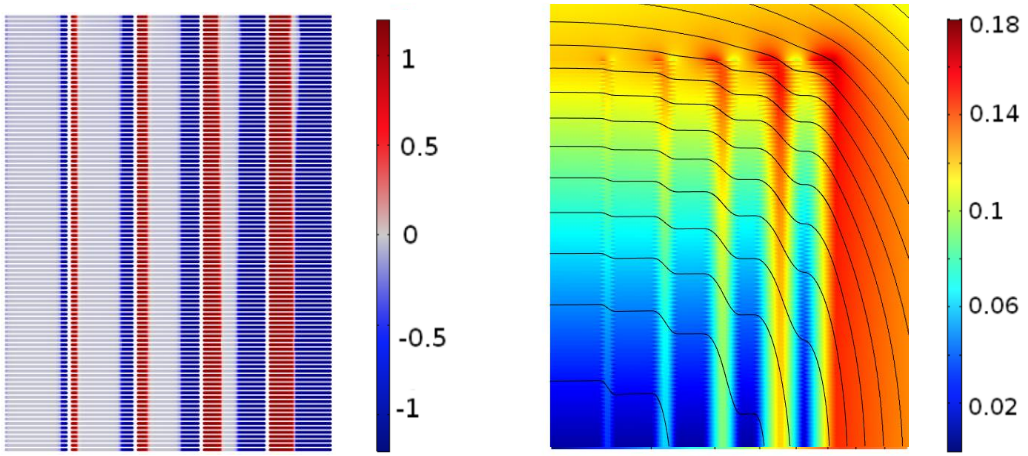
Iterative Multi-scaling and *H* Formulation





Iterative Multi-scaling and *H* Formulation

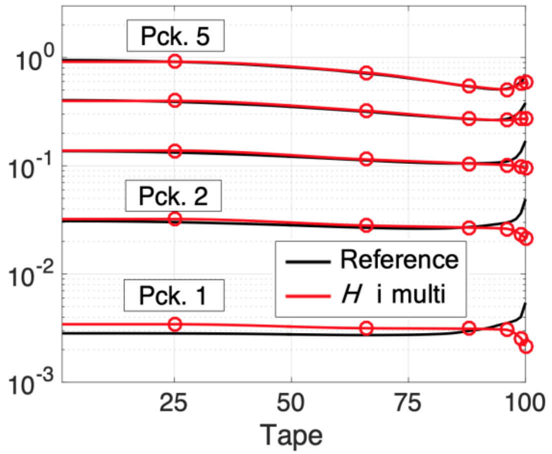




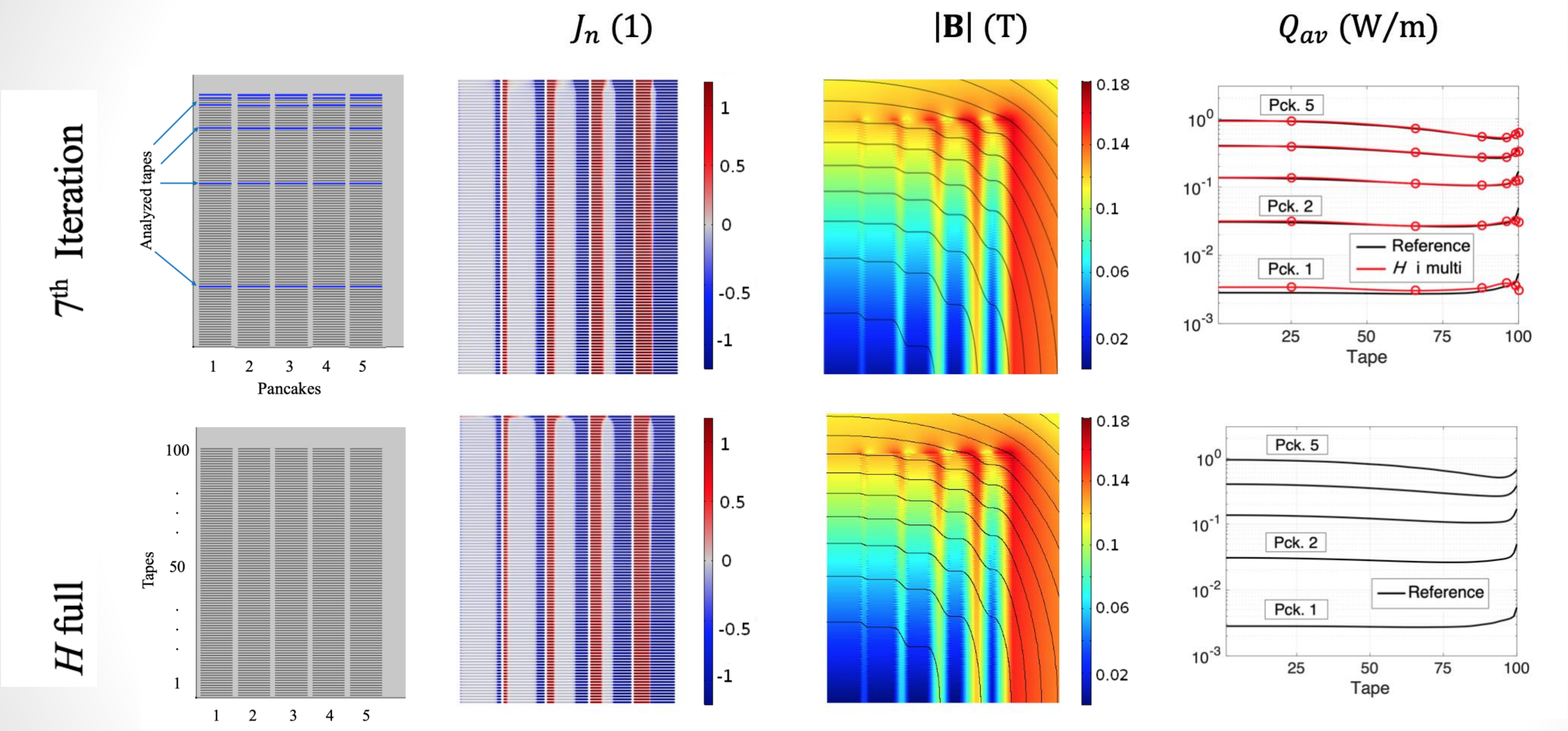
$$er_Q = -1.35\ \%$$

$$R^2 = 0.9661$$

$$\overline{ct} = 4.31\ \%$$



Case Study *H* Iterative Multi-scale Model



J coef. of det. $R^2 = 0.9803$ Losses error $er_Q = -0.56\ \%$ Normalized comp. time $\overline{ct} = 10.4\ \%$

Iterative Multi-scaling and 32 T All-Superconducting Magnet

IEEE TRANSACTIONS ON APPLIED SUPERCONDUCTIVITY, VOL. 28, NO. 3, APRIL 2018 4602005

Estimation of Losses in the (RE)BCO Two-Coil Insert of the NHMFL 32 T All-Superconducting Magnet

E. Berrospe-Juarez, V. M. R. Zermeño, F. Trillaud, A.V. Gavrilin, F. Grilli, D. V. Abraimov, D. K. Hilton, and H. W. Weijers

Abstract—**(RE)BCO commercial coated superconductors have been gaining an increasing interest due to the potentialities of using them in high-field magnets. The leading project is the 32 T user magnet recently tested successfully at full field at the National High Magnetic Field Laboratory (NHMFL), Tallahasee, FL, USA. This state-of-the-art high-field all-superconducting magnet, bath-cooled at 4.2 K, is comprised of a two-nested-coil insert pancake-wound with (RE)BCO tapes supplied by SuperPower Inc. and a multi-coil low temperature superconductors (LTS) outsert. To ensure the magnet's reliable operation, it is important to estimate the hysteresis losses. Such an estimate will allow implementing safe operational procedures to avoid premature quenching and, in the worst case scenario, insert failure. The insert coils have thousands of turns with notable variations in the critical current. Therefore, estimating the losses in such a large superconducting magnet presents a significant challenge. We propose here a new approach relying on a multiscale scheme to achieve a high computational efficiency. This new method is flexible enough to simulate different sections of the entire insert with the right level of detail while providing a larger computational speed than other approaches using the finite element method. Estimates of the hysteresis losses in the insert coils for a ramping operation sequence are presented.**

Index Terms—**AC losses, HTS magnets, (RE)BCO coils, finite element methods.**

I. Introduction

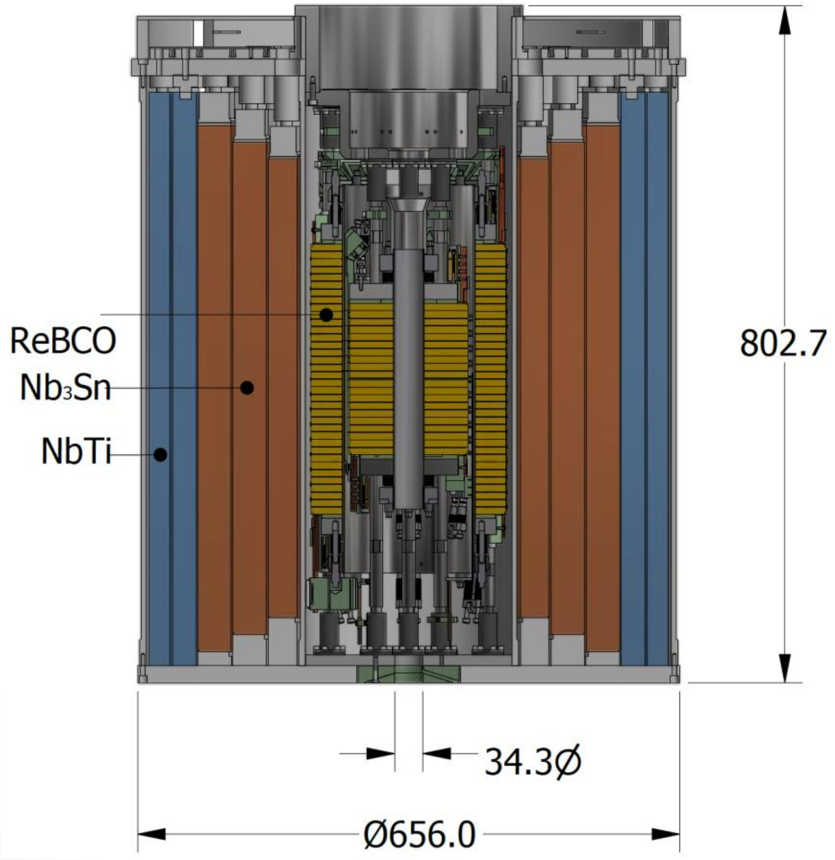
THREE decades after the discovery of the high temperature superconductors (HTS), the commercial (RE)BCO tapes have matured to the point that they can provide performances for high magnetic field applications. In particular, their high strength and remarkably high critical current are the key enablers to generate very high continuous magnetic fields that cannot be achieved with low temperature superconductors (LTS).

A new generation of high magnetic field user magnets is being developed at the National High Magnetic Field Laboratory (NHMFL) in an effort to increase its capability of providing magnetic fields above 30 T. The first magnet in the series is a 32-T all-superconducting one, which consists of a 15 T multi-coil LTS outsert custom-made by Oxford Instruments, Inc. and an innovative 17 T (RE)BCO-tape wound dual-coil insert operated at 4.2 K [1], [2]. The (RE)BCO tapes were supplied by SuperPower, Inc. It is the first magnet utilizing two (RE)BCO coils having thousands of turns. Besides minimizing the helium consumption, it is important to ensure the safe operation of the magnet during the changes in magnetic field required by the users. Thus, the risks associated with the technology should be clearly assessed. Any changes of magnetic field generated by the background LTS outsert and/or the HTS insert are expected to result in dissipations of stored energy absorbed by the cryogenic system.

Part of this assessment, a preliminary model was developed to quantify the losses in prototype coils of a few pancakes [3]. This model is based on the homogenization technique [4]. In the present work, a different, advanced approach is proposed to assess the hysteresis losses in the actual two-coil insert. The simulation that we performed enables us to estimate the self-field losses in the insert when the transport current in it is a triangle-shaped ramp cycle 2 min long, which does not repre-

NHMFL 32 T Superconducting Magnet

- The 32 T magnet is one-of-a-kind all superconduction magnet.
- The conducted analysis is focused in the HTS insert, at self field conditions.



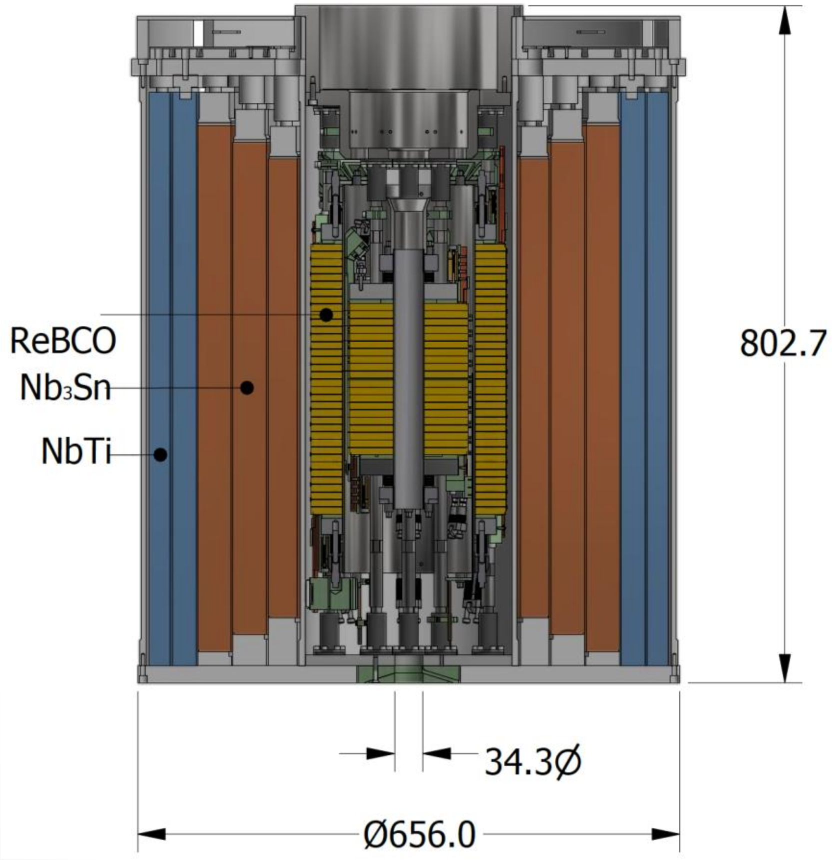
Sketch of the 32 T magnet from NHMFL [Xia et al., 2015].

32 T Insert Magnet Parameters

Parameter	Coil 1	Coil 2
Inner radius	20 mm	82 mm
Outer radius	70 mm	116 mm
Height	178 mm	320.4 mm
Pancakes	40	70
Turns/Pancake	253	145
HTS layer width	4 mm	4 mm
HTS layer thickness	1 µm	1 µm
Unit cell width	4.45 mm	4.45 mm
Unit cell thickness	197.63 µm	234.48 µm

NHMFL 32 T Superconducting Magnet

- The 32 T magnet is one-of-a-kind all superconduction magnet.
- The conducted analysis is focused in the HTS insert, at self field conditions.



Sketch of the 32 T magnet from NHMFL [Xia et al., 2015].

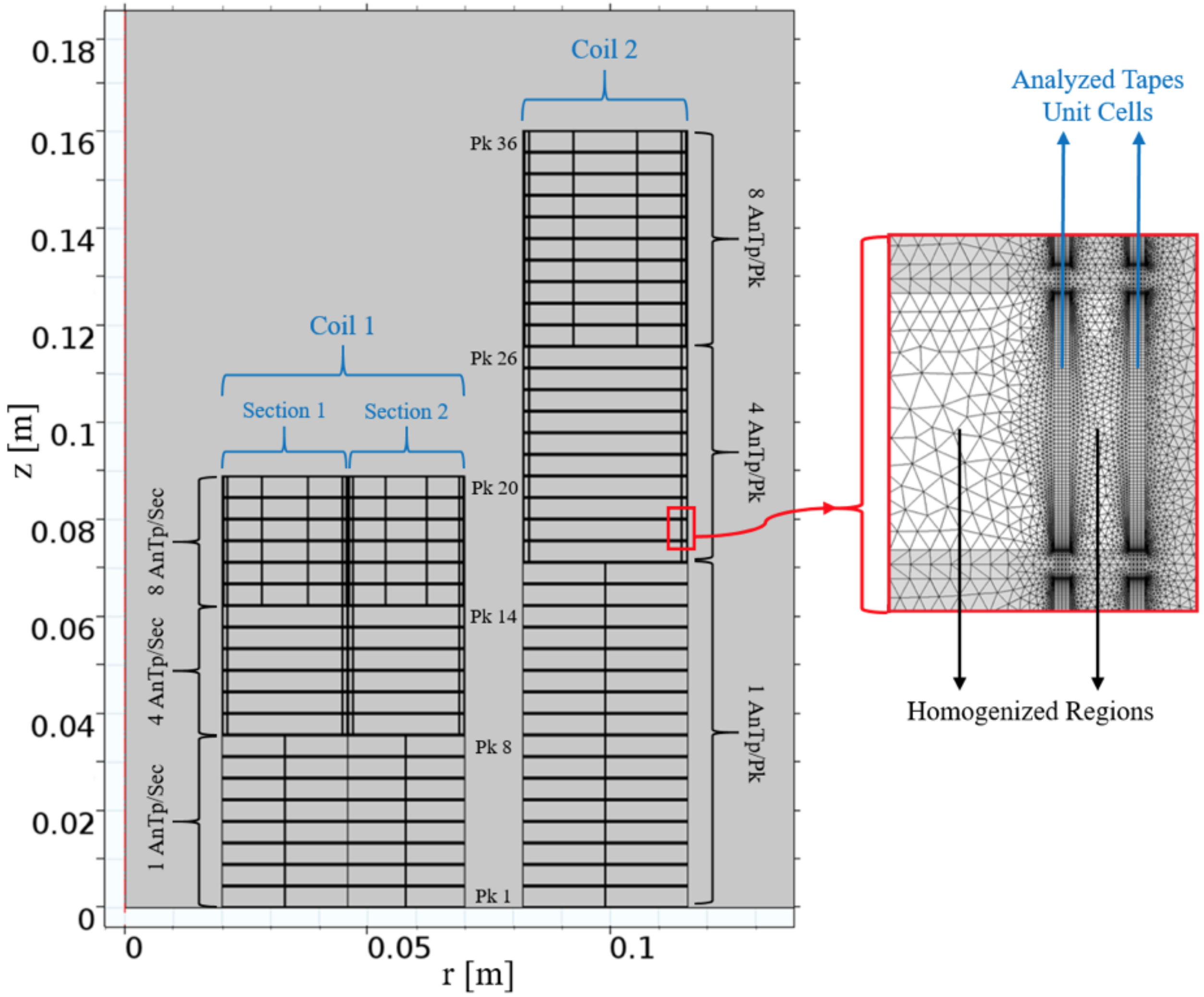
32 T Insert Magnet Parameters

Parameter	Coil 1	Coil 2
Inner radius	20 mm	82 mm
Outer radius	70 mm	116 mm
Height	178 mm	320.4 mm
Pancakes	40	70
Turns/Pancake	253	145
HTS layer width	4 mm	4 mm
HTS layer thickness	1 μm	1 μm
Unit cell width	4.45 mm	4.45 mm
Unit cell thickness	197.63 μm	234.48 μm

More than 20,000 turns

NHMFL 32 T Superconducting Magnet - Multi-scale Model

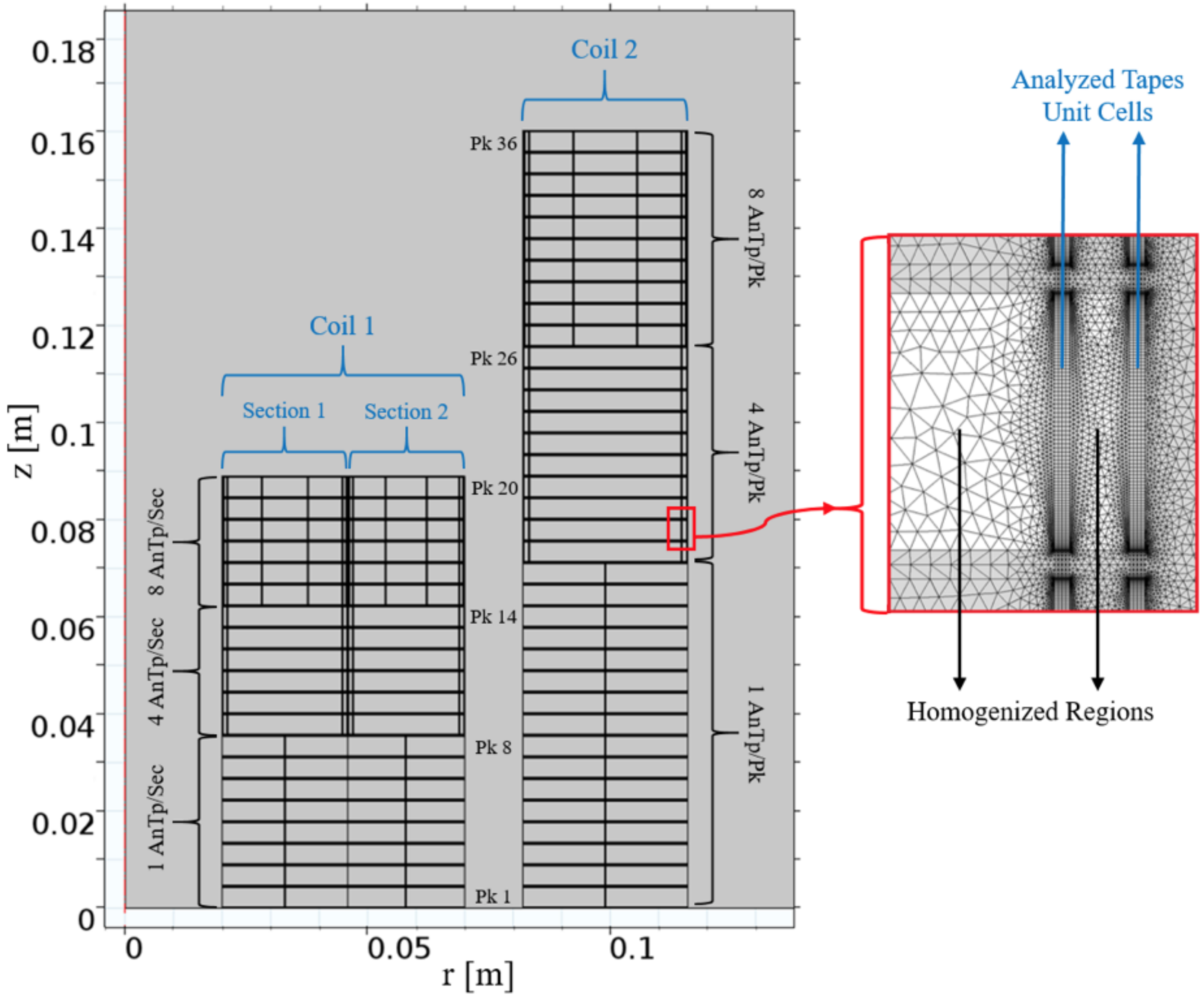
- The LTS outsert is not considered.
- The non-analyzed tapes are homogenized, a bulk region is considered.
- The more analyzed tapes are considered in the upper pancakes.



NHMFL 32 T Superconducting Magnet - Multi-scale Model

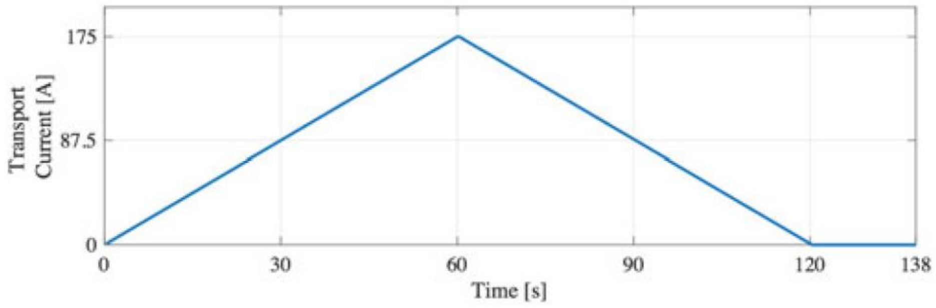
- The LTS outsert is not considered.
- The non-analyzed tapes are homogenized, a bulk region is considered.
- The more analyzed tapes are considered in the upper pancakes.
- The coefficient β allows to consider different J_c values in each section

$$J_c(B_r, B_z) = \frac{\beta \cdot J_{c0}}{\left(1 + \frac{\sqrt{k^2 B_z^2 + B_r^2}}{B_0}\right)^{\alpha}}$$



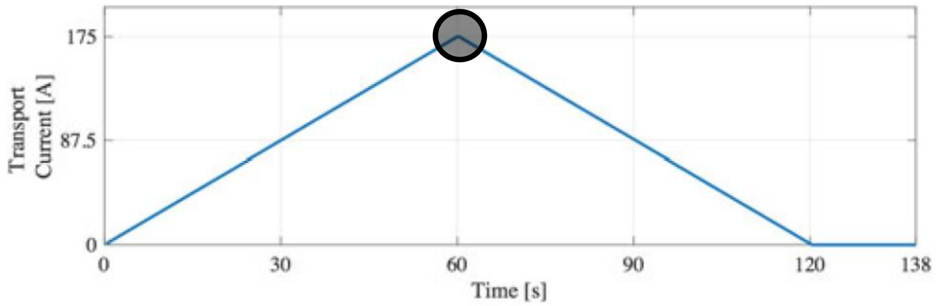
NHMFL 32 T Superconducting Magnet - Multi-scale Model

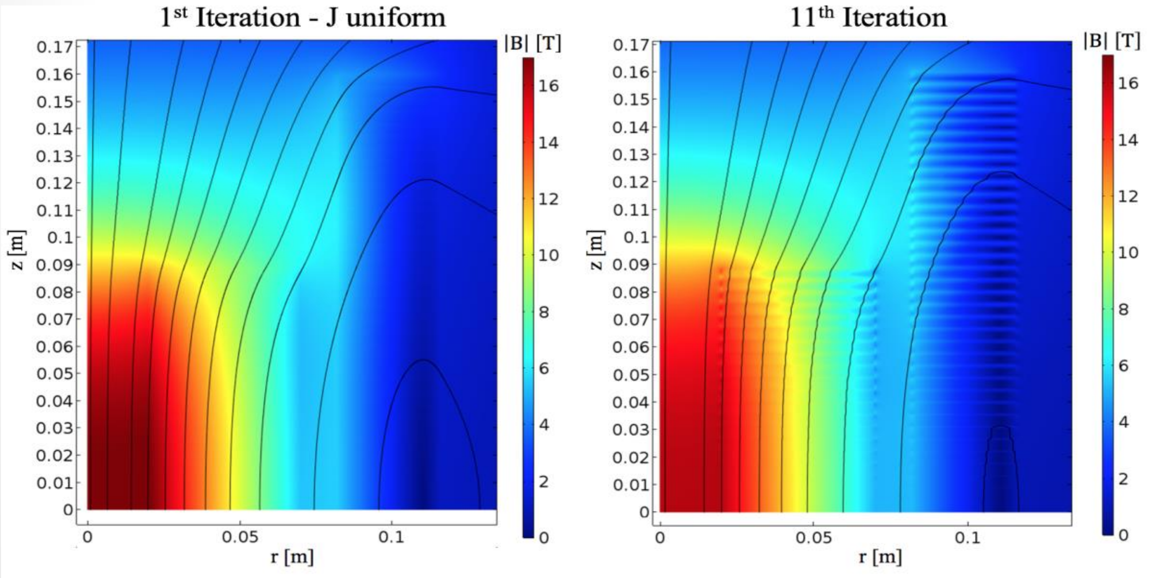
- The insert is charged linearly and subsequently discharged.



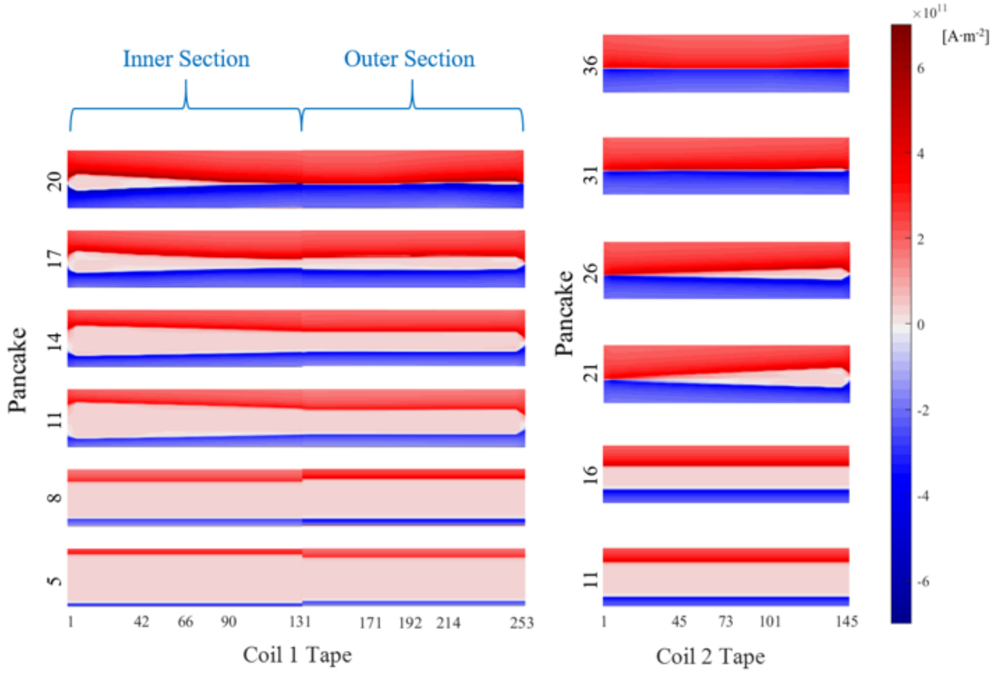
NHMFL 32 T Superconducting Magnet - Multi-scale Model

- The insert is charged linearly and subsequently discharged.



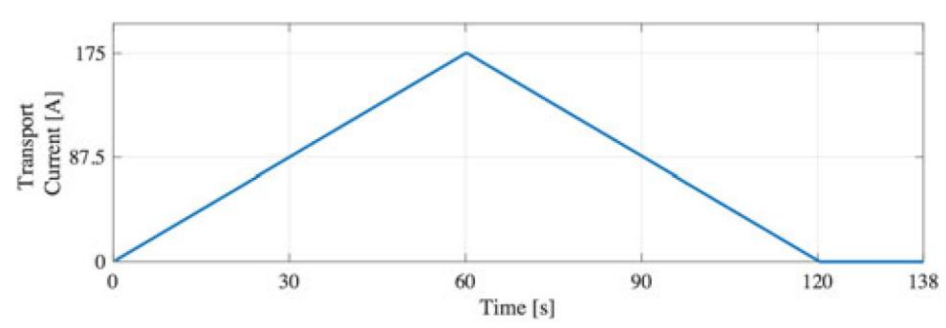


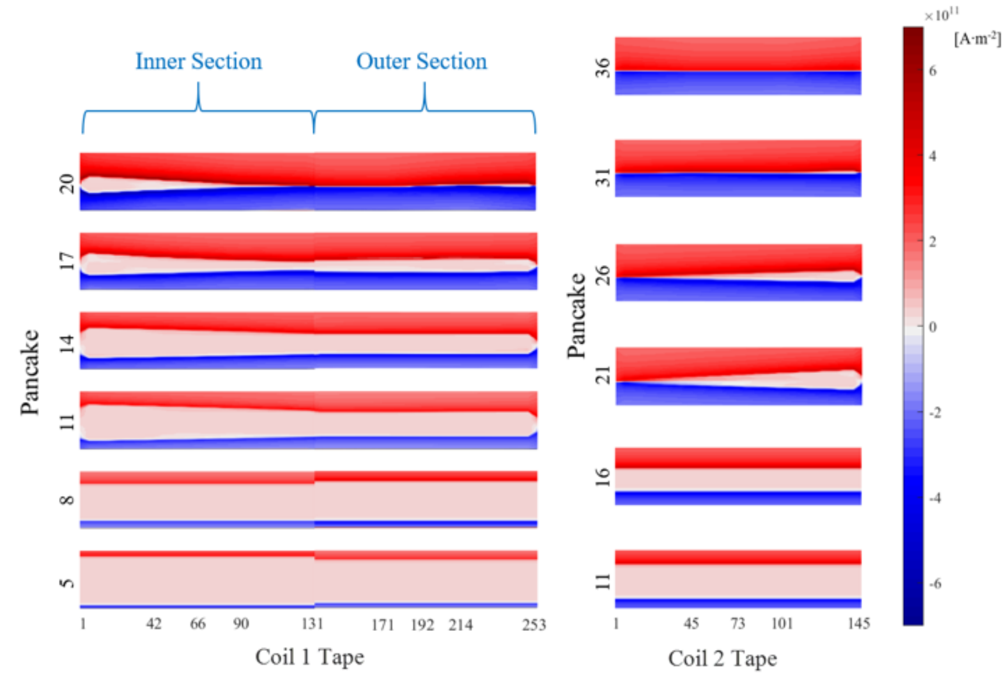
Magnetic field magnitude at peak current for the first and last iterations.



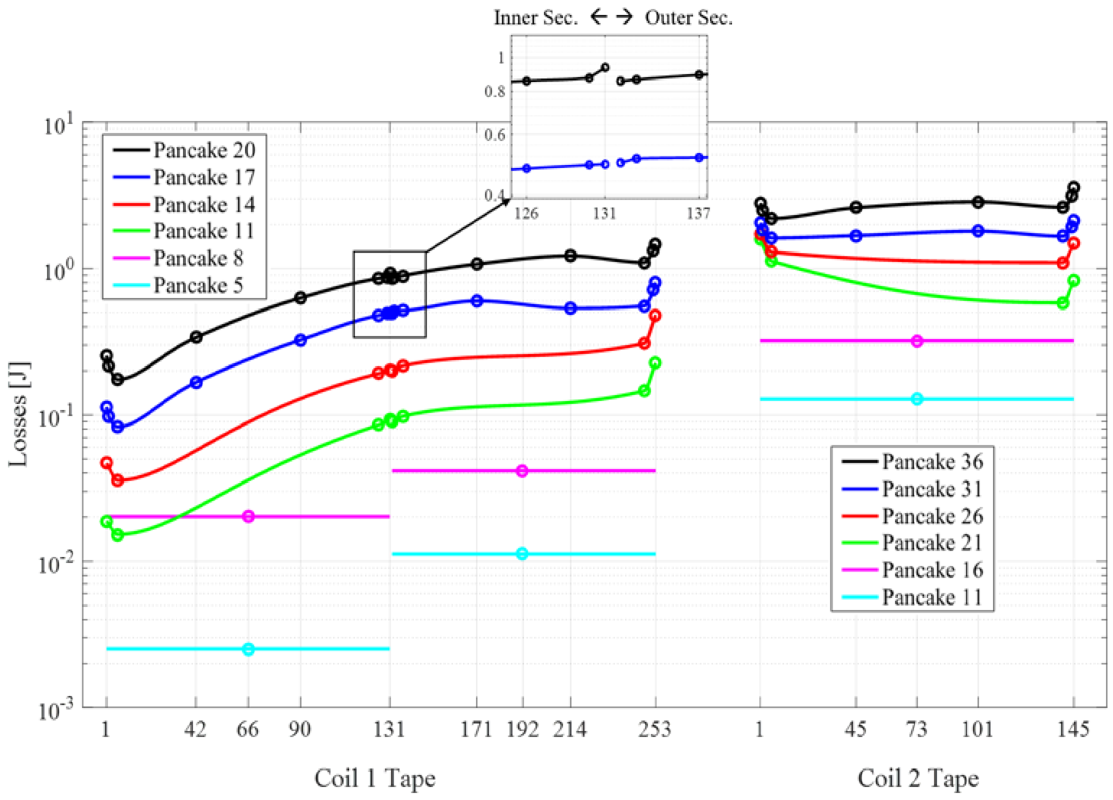
J at peak current in the last iterations. Just some pancakes are presented.

NHMFL 32 T Superconducting Magnet - Multi-scale Model





J at peak current in the last iterations. Just some pancakes are presented.



Losses in the 32 T insert.

T-*A* Formulation

T-A Formulation

- The *T-A* formulation was presented in [*Brambilla et al., 2006*] and [*Hong et al., 2006*].
- This strategy allows building mor efficient models of systems made of HTS tapes.

IOP Publishing

Superconductor Science and Technology

Supercond. Sci. Technol. **30** (2017) 024005 (7pp)

doi:10.1088/1361-6668/30/2/024005

An efficient 3D finite element method model based on the *T–A* formulation for superconducting coated conductors

Huiming Zhang[1], Min Zhang and Weijia Yuan

Department of Electronic and Electrical Engineering, University of Bath, Bath, BA2 7AY, UK

E-mail: zhanghuiming09@gmail.com and w.yuan@bath.ac.uk

Received 30 August 2016, revised 8 November 2016
Accepted for publication 18 November 2016
Published 13 December 2016

CrossMark

Abstract

An efficient three dimensional (3D) finite element method numerical model is proposed for superconducting coated conductors. The model is based on the *T–A* formulation and can be used to tackle 3D computational challenges for superconductors with high aspect ratios. By assuming a sheet approximation for the conductors, the model can speed up the computational process. The model has been validated by established analytical solutions. Two examples with complex geometries, which can hardly be simulated by the 2D model, are given. The model could be used to characterise and design large-scale applications using superconducting coated conductors, such as high field magnets and other electrical devices.

Keywords: superconducting coated conductor, FEM, *T–A* formulation, efficient

(Some figures may appear in colour only in the online journal)

1. Introduction

Second generation high temperature superconductors (2G

dimensional reduction, it is very difficult to set the boundary imposing the transport current [8]. The transport current is typically imposed through a cross-sectional slice of the con-

JOURNAL OF APPLIED PHYSICS **122**, 043903 (2017)

CrossMark

A finite element model for simulating second generation high temperature superconducting coils/stacks with large number of turns

Fei Liang,[1] Sriharsha Venuturumilli,[1] Huiming Zhang,[1] Min Zhang,[1] Jozef Kvitkovic,[2] Sastry Pamidi,[2] Yawei Wang,[1] and Weijia Yuan[1,a)]

[1]*Department of Electronic and Electrical Engineering, University of Bath, Bath BA2 7AY, United Kingdom*
[2]*Center for Advanced Power Systems, Florida State University, Tallahassee, Florida 32310, USA*

(Received 25 May 2017; accepted 12 July 2017; published online 27 July 2017)

An efficient two dimensional T-A formulation based approach is proposed to calculate the electromagnetic characteristics of tape stacks and coils made of second generation high temperature superconductors. In the approach, a thin strip approximation of the superconductor is used in which the superconducting layer is modeled as a 1-dimensional domain. The formulation is mainly based on the calculation of the current vector potential ***T*** in the superconductor layer and the calculation of the magnetic vector potential ***A*** in the whole space, which are coupled together in the model. Compared with previous T-based models, the proposed model is innovative in terms of magnetic vector potential A solving, which is achieved by using the differential method, instead of the integral method. To validate the T-A formulation model, it is used to simulate racetrack coils made of second generation high temperature superconducting (2G HTS) tape, and the results are compared with the experimentally obtained data on the AC loss. The results show that the T-A formulation is accurate and efficient in calculating 2G HTS coils, including magnetic field distribution, current density distribution, and AC loss. Finally, the proposed model is used for simulating a 2000 turn coil to demonstrate its effectiveness and efficiency in simulating large-scale 2G HTS coils. *Published by AIP Publishing.* [http://dx.doi.org/10.1063/1.4995802]

I. INTRODUCTION

Since their discovery in 1986, high temperature superconductors (HTS) have attracted considerable attention from both academia and industry.[1] Specifically, in recent years, commercialization of second generation high temperature superconducting tapes has brought the large-scale and widespread applications of high temperature superconductors to reality. Till now, second generation HTS tapes have been used for building various kinds of superconducting devices such as fault current limiters, power cables, and energy storage systems, some of them are being operated in live power grids.[2,3] High field magnets for high energy physics and biomedical research application are also being explored.[4,5]

ellipse, rectangular, cruciform, and box.[6] Brandt extended the Norris model to the electromagnetic calculation of a type-II superconductor strip.[7] Regarding the integral method, Clem first proposed a theoretical framework for estimating the AC loss in a finite Z stack by using anisotropic homogeneous-medium approximation.[8] Yuan *et al.* extended the model by replacing the frontier with a quadratic function and introducing the magnetic field dependence of J_c.[9–11] Prigozhin proposed a free-marching numerical scheme based on magnetic vector potential formulation, which can be used for numerical solution of critical-state problems with arbitrary current-voltage laws.[12] Subsequently, Pardo *et al.* simulated the electromagnetic properties and AC losses of pancake coils and

T-A Formulation

- The *T-A* formulation is implemented by the combination of the *T* and the *A* formulations.

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{J} = \nabla \times \mathbf{T}$$

T-A Formulation

- The *T-A* formulation is implemented by the combination of the *T* and the *A* formulations.

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{J} = \nabla \times \mathbf{T}$$

$$\nabla \times \mathbf{H} = \mathbf{J} \qquad \nabla \times \mathbf{E} = -\frac{\partial(\mathbf{B})}{\partial t}$$

T-A Formulation

- The *T-A* formulation is implemented by the combination of the *T* and the *A* formulations.

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{J} = \nabla \times \mathbf{T}$$

$$\nabla \times \mathbf{H} = \mathbf{J} \qquad \nabla \times \mathbf{E} = -\frac{\partial(\mathbf{B})}{\partial t}$$

$$\downarrow \qquad \downarrow$$

$$\mathbf{B} = \mu\mathbf{H} \qquad \rho\mathbf{J} = \mathbf{E}$$

T-A Formulation

- The *T-A* formulation is implemented by the combination of the *T* and the *A* formulations.

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{J} = \nabla \times \mathbf{T}$$

$$\nabla \times \mathbf{H} = \mathbf{J}$$

$$\nabla \times \mathbf{E} = -\frac{\partial(\mathbf{B})}{\partial t}$$

$$\downarrow$$

$$\mathbf{B} = \mu\mathbf{H}$$

$$\rho\mathbf{J} = \mathbf{E}$$

$$\downarrow$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{J} = \nabla \times \mathbf{T}$$

T-A Formulation

- The *T-A* formulation is implemented by the combination of the *T* and the *A* formulations.

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{J} = \nabla \times \mathbf{T}$$

$$\nabla \times \mathbf{H} = \mathbf{J} \qquad \nabla \times \mathbf{E} = -\frac{\partial(\mathbf{B})}{\partial t}$$

$$\downarrow \qquad \downarrow$$

$$\mathbf{B} = \mu\mathbf{H} \qquad \rho\mathbf{J} = \mathbf{E}$$

$$\downarrow \qquad \downarrow$$

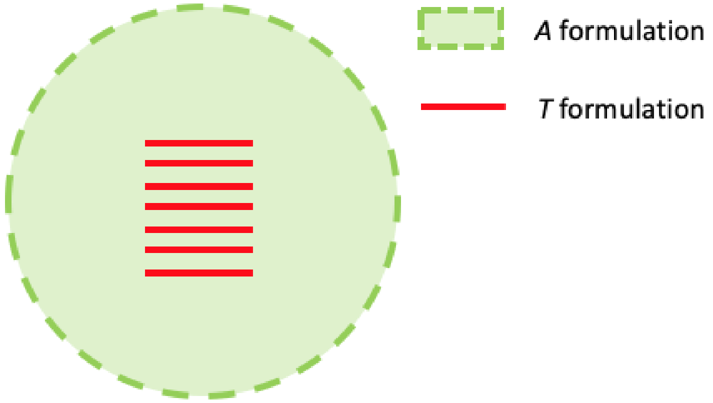
$$\mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{J} = \nabla \times \mathbf{T}$$

$$\downarrow \qquad \downarrow$$

$$\nabla \times \nabla \times \mathbf{A} = \mu\mathbf{J} \qquad \nabla \times \rho\nabla \times \mathbf{T} = -\frac{\partial(\mathbf{B})}{\partial t}$$

T-A Formulation

- The *T-A* formulation is implemented by the combination of the *T* and the *A* formulations.
- *A* is defined all over the bounded universe, while *T* is exclusively defined along the superconducting medium.



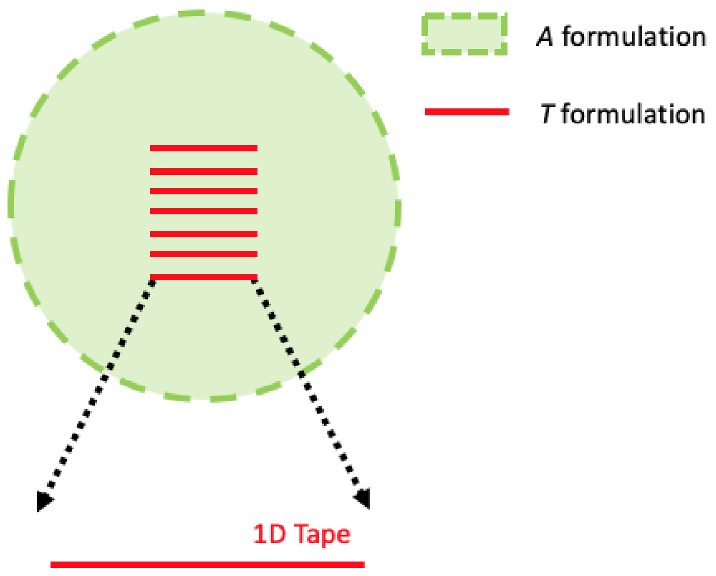
$$\mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{J} = \nabla \times \mathbf{T}$$

$$\nabla \times \nabla \times \mathbf{A} = \mu \mathbf{J} \qquad \nabla \times \rho \nabla \times \mathbf{T} = -\frac{\partial (\mathbf{B})}{\partial t}$$

T-A Formulation

- The *T-A* formulation is implemented by the combination of the *T* and the *A* formulations.
- *A* is defined all over the bounded universe, while *T* is exclusively defined along the superconducting medium.
- The superconducting layer of the tapes are modelled as one dimensional (1D) objects.

$$\mathbf{B} = \nabla \times \mathbf{A} \qquad \mathbf{J} = \nabla \times \mathbf{T}$$



$$\nabla \times \nabla \times \mathbf{A} = \mu \mathbf{J} \qquad \nabla \times \rho \nabla \times \mathbf{T} = -\frac{\partial(\mathbf{B})}{\partial t}$$

T-A Formulation

- The *T-A* formulation is implemented by the combination of the *T* and the *A* formulations.
- *A* is defined all over the bounded universe, while *T* is exclusively defined along the superconducting medium.
- The superconducting layer of the tapes are modelled as one dimensional (1D) objects.

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$B_x = \frac{\partial A_z}{\partial y} \qquad B_y = -\frac{\partial A_z}{\partial x}$$

$$\mathbf{J} = \nabla \times \mathbf{T}$$

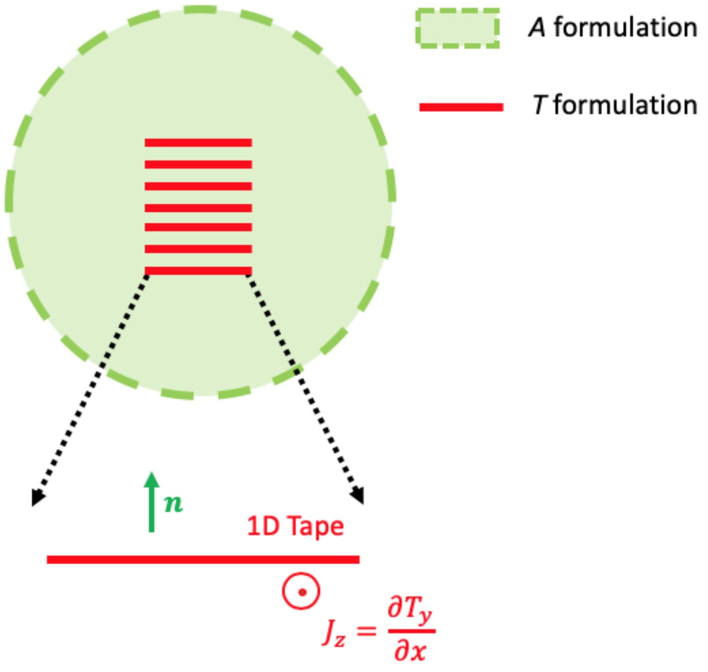
$$J_z = \frac{\partial T_y}{\partial x}$$

$$\nabla \times \nabla \times \mathbf{A} = \mu \mathbf{J}$$

$$\nabla^2 A_z = -\mu\, J_z$$

$$\nabla \times \rho \nabla \times \mathbf{T} = -\frac{\partial (\mathbf{B})}{\partial t}$$

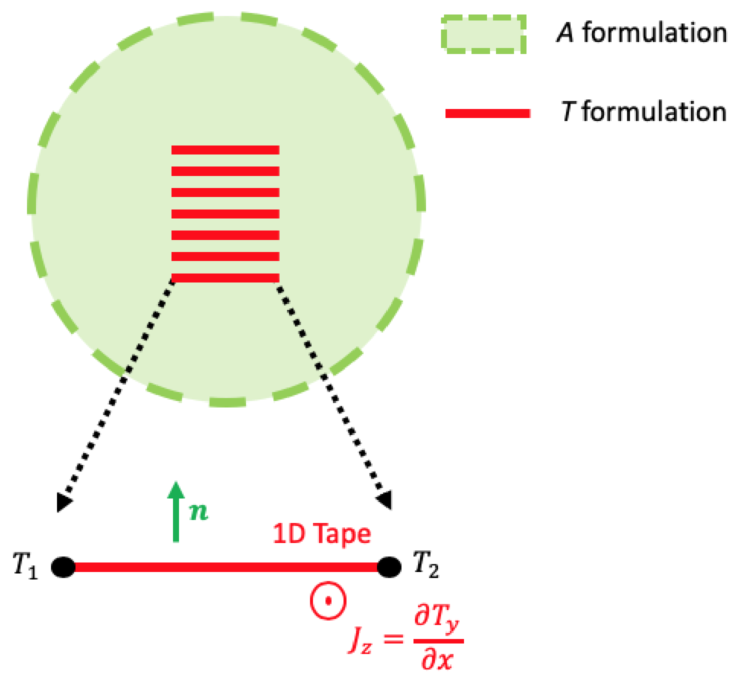
$$\frac{\partial}{\partial x}\left(\rho_{HTS} \frac{\partial T_y}{\partial x}\right) = \frac{\partial B_y}{\partial t}$$



T-A Formulation

- The *T-A* formulation is implemented by the combination of the *T* and the *A* formulations.
- *A* is defined all over the bounded universe, while *T* is exclusively defined along the superconducting medium.
- The superconducting layer of the tapes are modelled as one dimensional (1D) objects.
- The transport current is imposed by means of the boundary conditions for *T*.

$$I = (T_1 - T_2)\delta$$

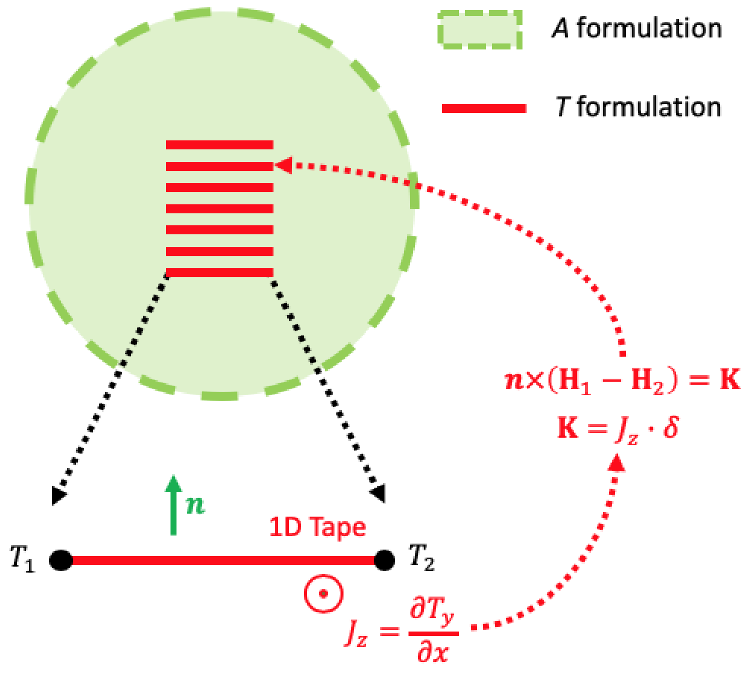


T-A Formulation

- The *T-A* formulation is implemented by the combination of the *T* and the *A* formulations.
- *A* is defined all over the bounded universe, while T is exclusively defined along the superconducting medium.
- The superconducting layer of the tapes are modelled as one dimensional (1D) objects.
- The transport current is imposed by means of the boundary conditions for *T*.

$$I = (T_1 - T_2)\delta$$

- The surface current density *K* is impressed into the *A* formulation by means of a Neumann boundary condition.



Multi-scaling, Homogenization and *T-A* Formulation

IOP Publishing   Superconductor Science and Technology

Supercond. Sci. Technol. **32** (2019) 065003 (14pp)   https://doi.org/10.1088/1361-6668/ab0d66

Real-time simulation of large-scale HTS systems: multi-scale and homogeneous models using the *T–A* formulation

Edgar Berrospe-Juarez[1], Víctor M R Zermeño[2], Frederic Trillaud[3] and Francesco Grilli[4]

[1] Postgraduate School of Engineering, National Autonomous University of México, Mexico
[2] NKT, Germany
[3] Institute of Engineering, National Autonomous University of México, Mexico
[4] Karlsruhe Institute of Technology, Germany

E-mail: eberrospej@iingen.unam.mx

Received 27 November 2018, revised 27 February 2019
Accepted for publication 6 March 2019
Published 30 April 2019

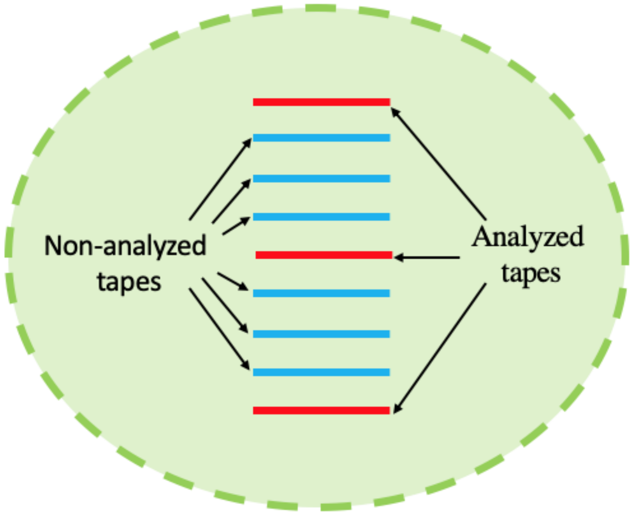
CrossMark

Abstract

The emergence of second-generation high temperature superconducting (HTS) tapes has favored the development of large-scale superconductor systems. The mathematical models capable of estimating electromagnetic quantities in superconductors have evolved from simple analytical models to complex numerical models. The available analytical models are limited to the analysis of single wires or infinite arrays that, in general, do not represent actual devices in real applications. The numerical models based on the finite element method using the *H* formulation of Maxwell's equations are useful for the analysis of medium-size systems, but their application in large-scale systems is problematic due to the excessive computational cost in terms of memory and computation time. Therefore it is necessary to devise new strategies to make the computation

Multi-scaling and *T-A* Formulation

- The *T-A* multi-scale models consider a reduced number of analyzed tapes.

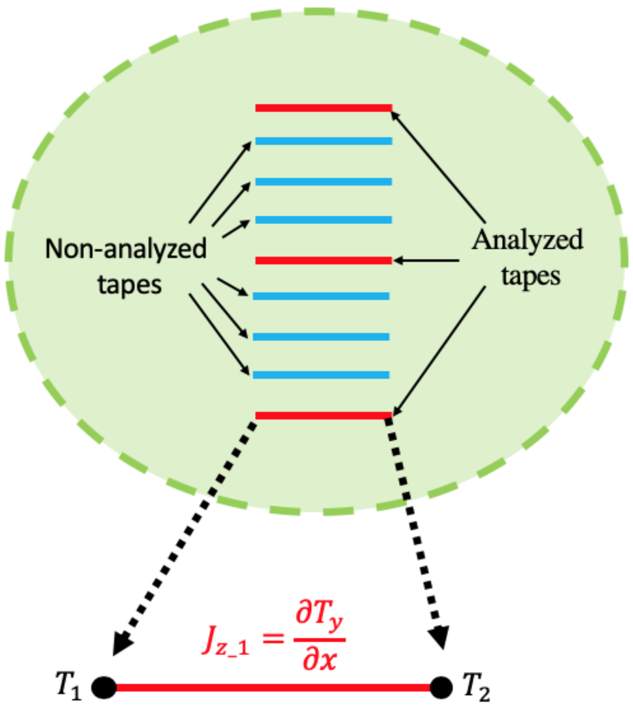


Multi-scaling and *T-A* Formulation

- The *T-A* multi-scale models consider a reduced number of analyzed tapes.
- *T* is exclusively defined along the analyzed tapes.

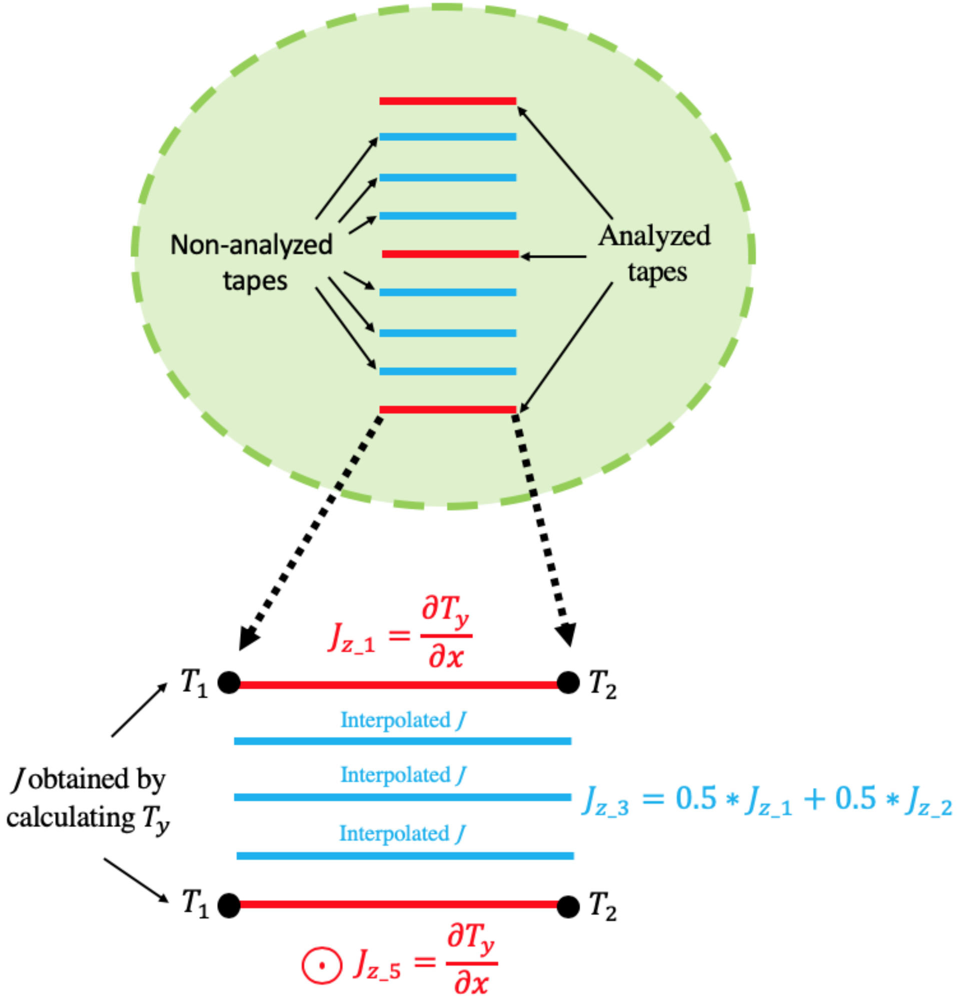
$$\nabla^2 A_z = -\mu J_z$$

$$\frac{\partial}{\partial x}\left(\rho_{HTS}\frac{\partial T_y}{\partial x}\right) = \frac{\partial B_y}{\partial t}$$



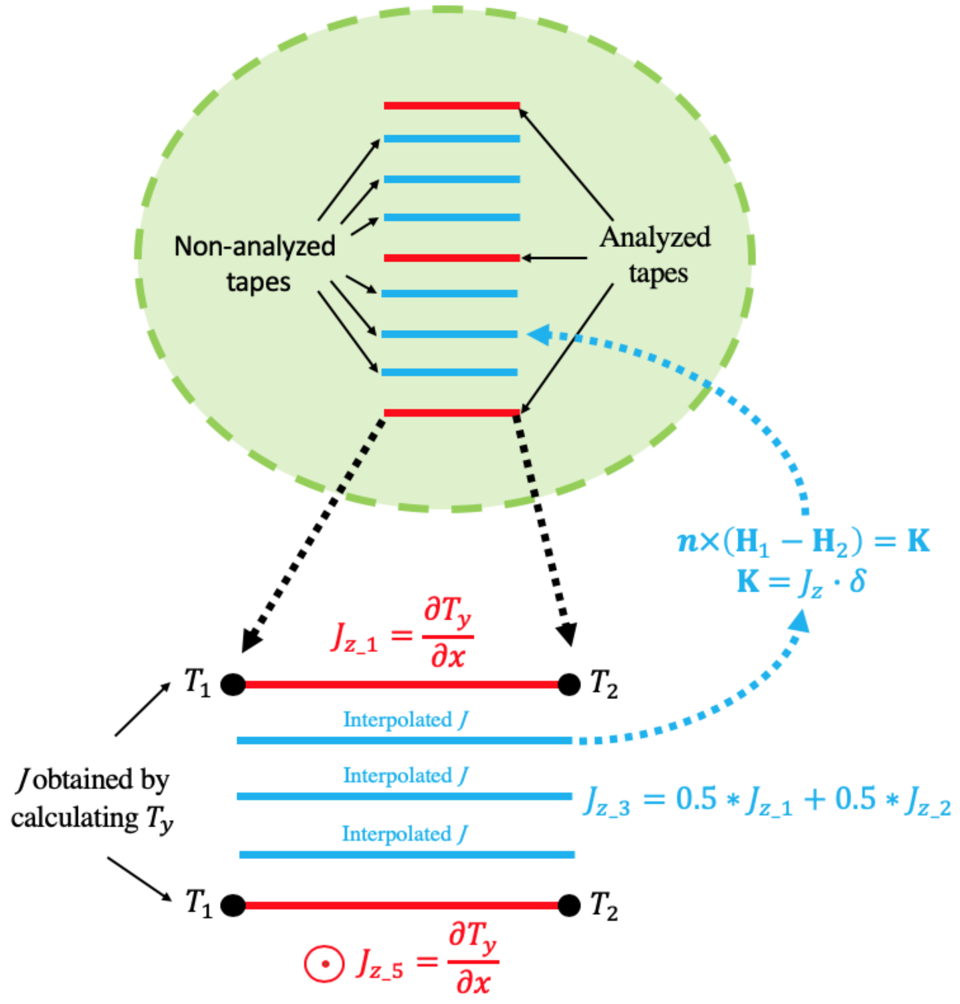
Multi-scaling and *T-A* Formulation

- The *T-A* multi-scale models consider a reduced number of analyzed tapes.
- *T* is exclusively defined along the analyzed tapes.
- The J in the non-analyzed tapes is approximated by interpolation.



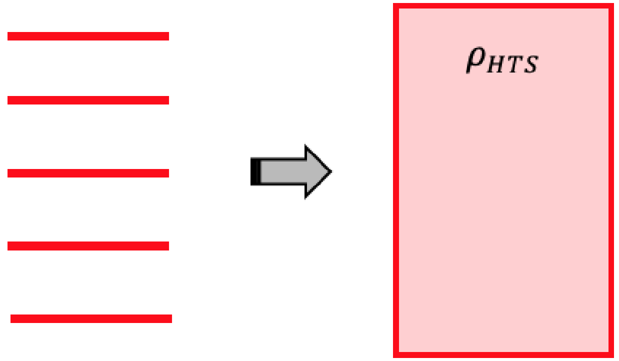
Multi-scaling and *T-A* Formulation

- The *T-A* multi-scale models consider a reduced number of analyzed tapes.
- *T* is exclusively defined along the analyzed tapes.
- The J in the non-analyzed tapes is approximated by interpolation.
- The *T-A* formulation allows the simultaneous computation of *T* and *A*, then it is not necessary to implement an iterative algorithm.



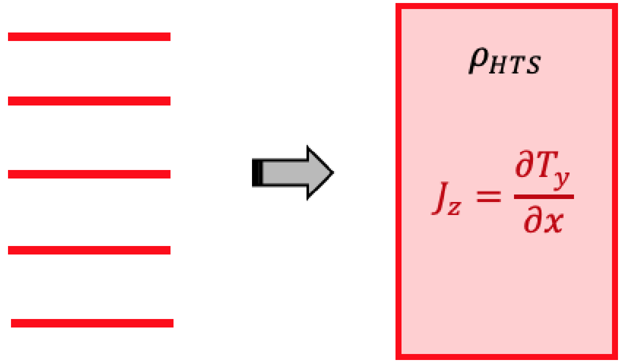
Homogenization and *T-A* Formulation

- The homogenizations tranforms a HTS tapes stack into an anisotropic bulk.



Homogenization and *T-A* Formulation

- The homogenizations tranforms a HTS tapes stack into an anisotropic bulk.
- *T* is exclusively defined inside the bulk.
- The influence of B_x in T is neglected, therefore T is forced to have only one component.



$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$B_x = \frac{\partial A_z}{\partial y} \qquad B_y = -\frac{\partial A_z}{\partial x}$$

$$\mathbf{J} = \nabla \times \mathbf{T}$$

$$J_z = \frac{\partial T_y}{\partial x}$$

$$\nabla \times \nabla \times \mathbf{A} = \mu \mathbf{J}$$

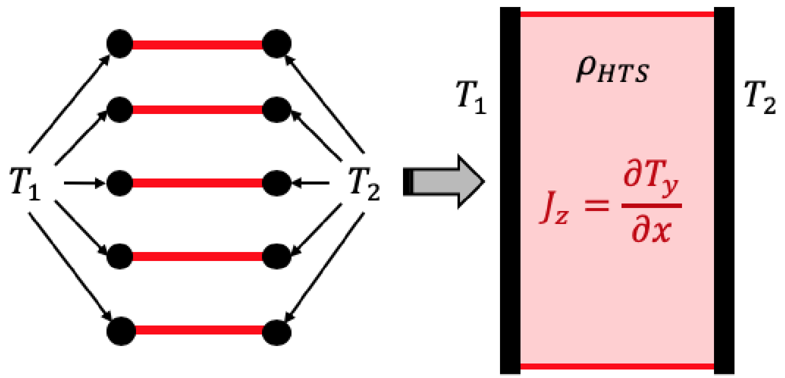
$$\nabla^2 A_z = -\mu J_z$$

$$\nabla \times \rho \nabla \times \mathbf{T} = -\frac{\partial (\mathbf{B})}{\partial t}$$

$$\frac{\partial}{\partial x}\left(\rho_{HTS} \frac{\partial T_y}{\partial x}\right) = \frac{\partial B_y}{\partial t}$$

Homogenization and *T-A* Formulation

- The homogenizations tranforms a HTS tapes stack into an anisotropic bulk.
- *T* is exclusively defined inside the bulk.
- The influence of B_x in T is neglected, therefore T is forced to have only one component.
- The boundary conditions force that each infinitesimal tape transport the same current of its original counterpart.

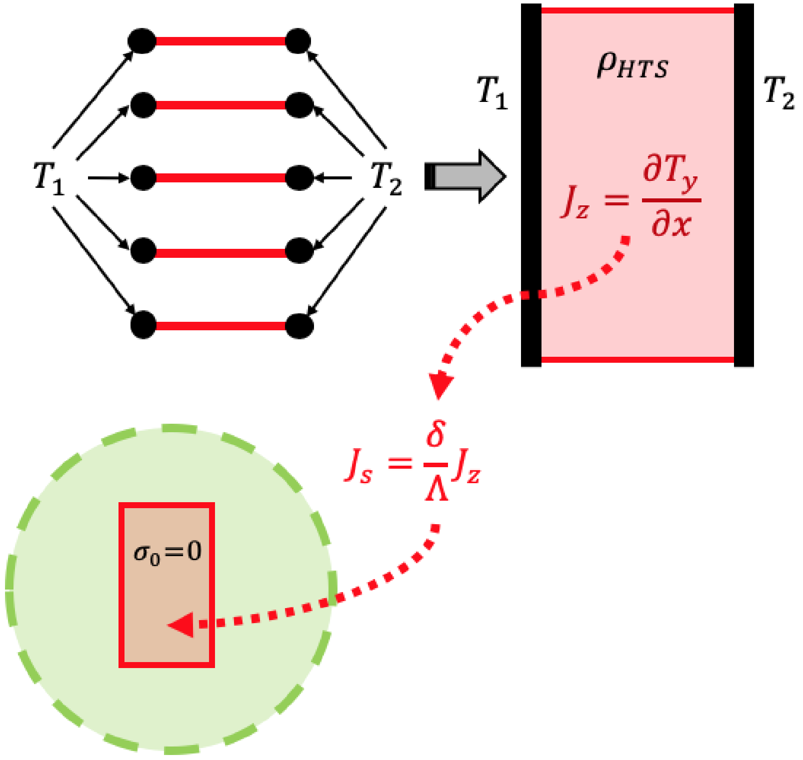


$$\nabla^2 A_z = -\mu J_z$$

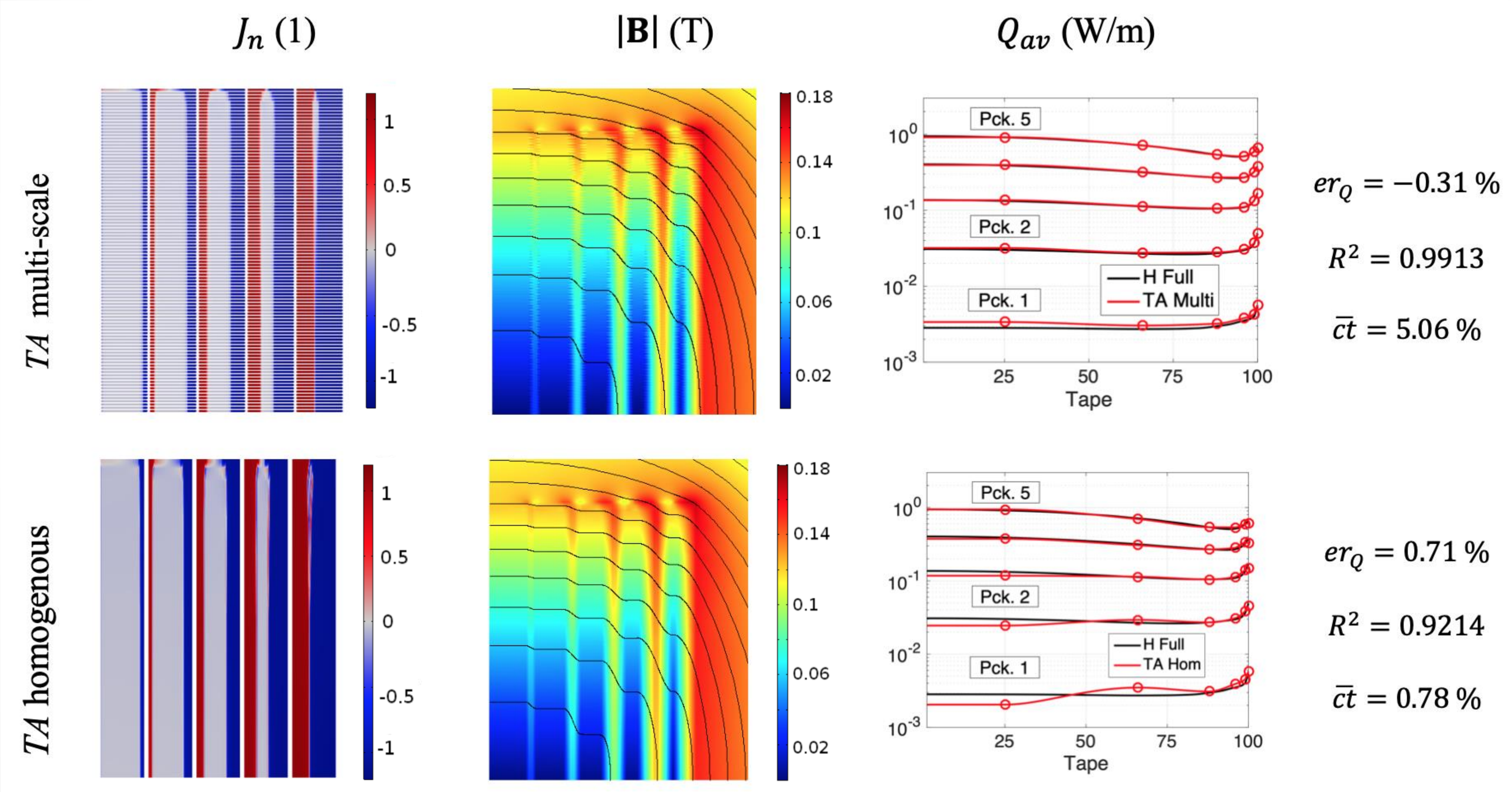
$$\frac{\partial}{\partial x}\left(\rho_{HTS}\frac{\partial T_y}{\partial x}\right) = \frac{\partial B_y}{\partial t}$$

Homogenization and *T-A* Formulation

- The homogenizations tranforms a HTS tapes stack into an anisotropic bulk.
- T is exclusively defined inside the bulk.
- The influence of B_x in T is neglected, therefore T is forced to have only one component.
- The boundary conditions force that each infinitesimal tape transport the same current of its original counterpart.
- The J_z inside the bulk is scale to be impressed as an external source.



Case Study *TA* Multi-scale and *TA* Homogeneous Models



Case Study Models Comparison

Table. Models comparison.

H full model Reference	Losses (W/m)		Computation time (h)
	127.24		31 h 32 min
Model	er_Q (%)	R^2 (1)	$\overline{ct}$ (%)
H multi-scale	-21.7	0.0304	1.45
H iterative multi-scale	-0.56	0.9803	10.51
H homogenous	1.28	0.9221	1.94
TA full	0.64	0.9922	10.25
TA simultaneous multi-scale	-0.31	0.9913	5.06
TA homogenous	0.71	0.9214	0.78

Case Study Models Comparison

Table. Models comparison.

H full model	Losses (W/m)		Computation time (h)
Reference	127.24		31 h 32 min
Model	er_Q (%)	R^2 (1)	$\overline{ct}$ (%)
H multi-scale	-21.7	0.0304	1.45
H iterative multi-scale	-0.56	0.9803	10.51
H homogenous	1.28	0.9221	1.94
TA full	0.64	0.9922	10.25
TA simultaneous multi-scale	-0.31	0.9913	5.06
TA homogenous	0.71	0.9214	0.78

Multi-scaling, Homogenization and *T-A* Formulation

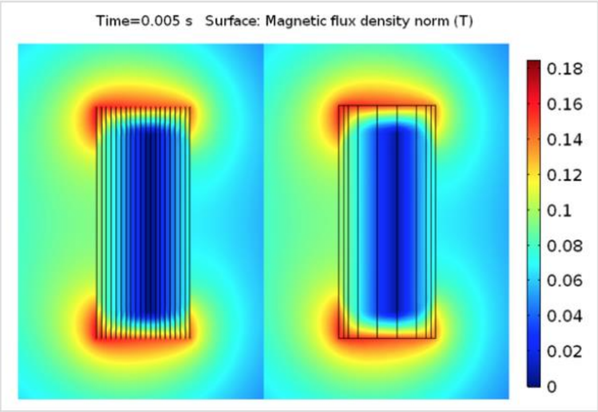
- Small examples of the code are available on-line. http://www.htsmodelling.com/

HTS MODELLING WORKGROUP | Modelling of High Temperature Superconductors (HTS)

T-A multi-scale and homogeneous models for the *Benchmark #3* (shared by Edgar Berrospe, National Autonomous University of Mexico, Mexico). These two models address the analysis of the Benchmark #3, a 20 HTS tapes stack. The models show how the multi-scale and homogeneous methods are adapted to be used in conjunction with the T-A formulation. The achieved simplification in the description of the system allow to reduce the computation time.

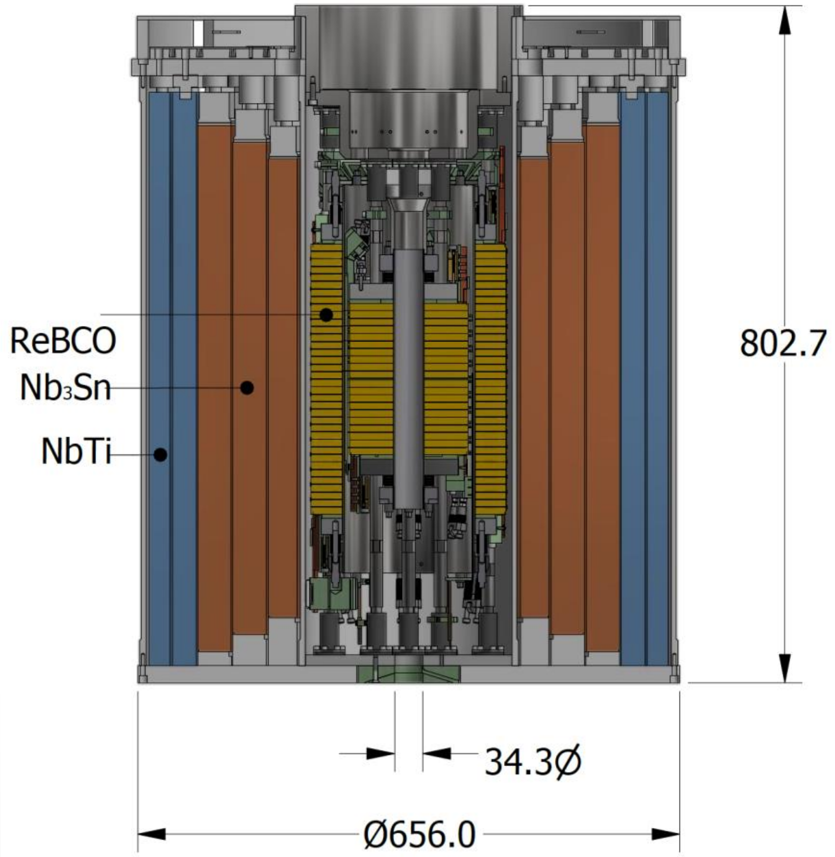
Comsol files (version 5.2a): here.

Reference article: Edgar Berrospe-Juarez et al 2019 Supercond. Sci. Technol. 32 065003



NHMFL 32 T Superconducting Magnet

- The 32 T magnet is one-of-a-kind all superconduction magnet.
- The conducted analysis is focused in the HTS insert, at self field conditions.



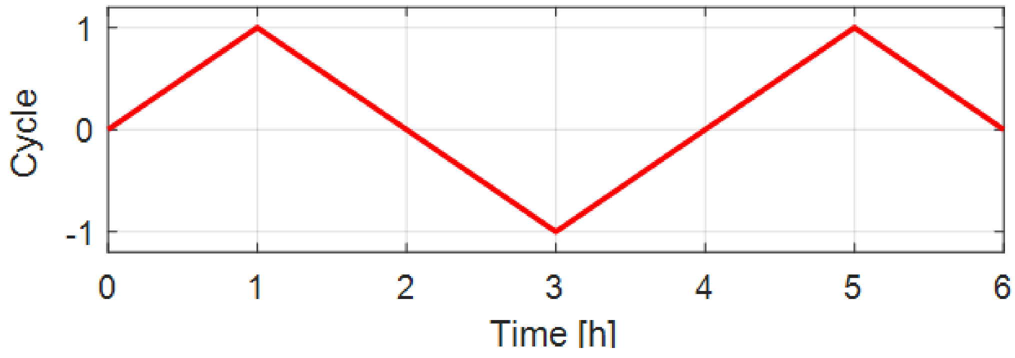
Sketch of the 32 T magnet from NHMFL [Xia et al., 2015].

32 T Insert Magnet Parameters

Parameter	Coil 1	Coil 2
Inner radius	20 mm	82 mm
Outer radius	70 mm	116 mm
Height	178 mm	320.4 mm
Pancakes	40	70
Turns/Pancake	253	145
HTS layer width	4 mm	4 mm
HTS layer thickness	1 μm	1 μm
Unit cell width	4.45 mm	4.45 mm
Unit cell thickness	197.63 μm	234.48 μm

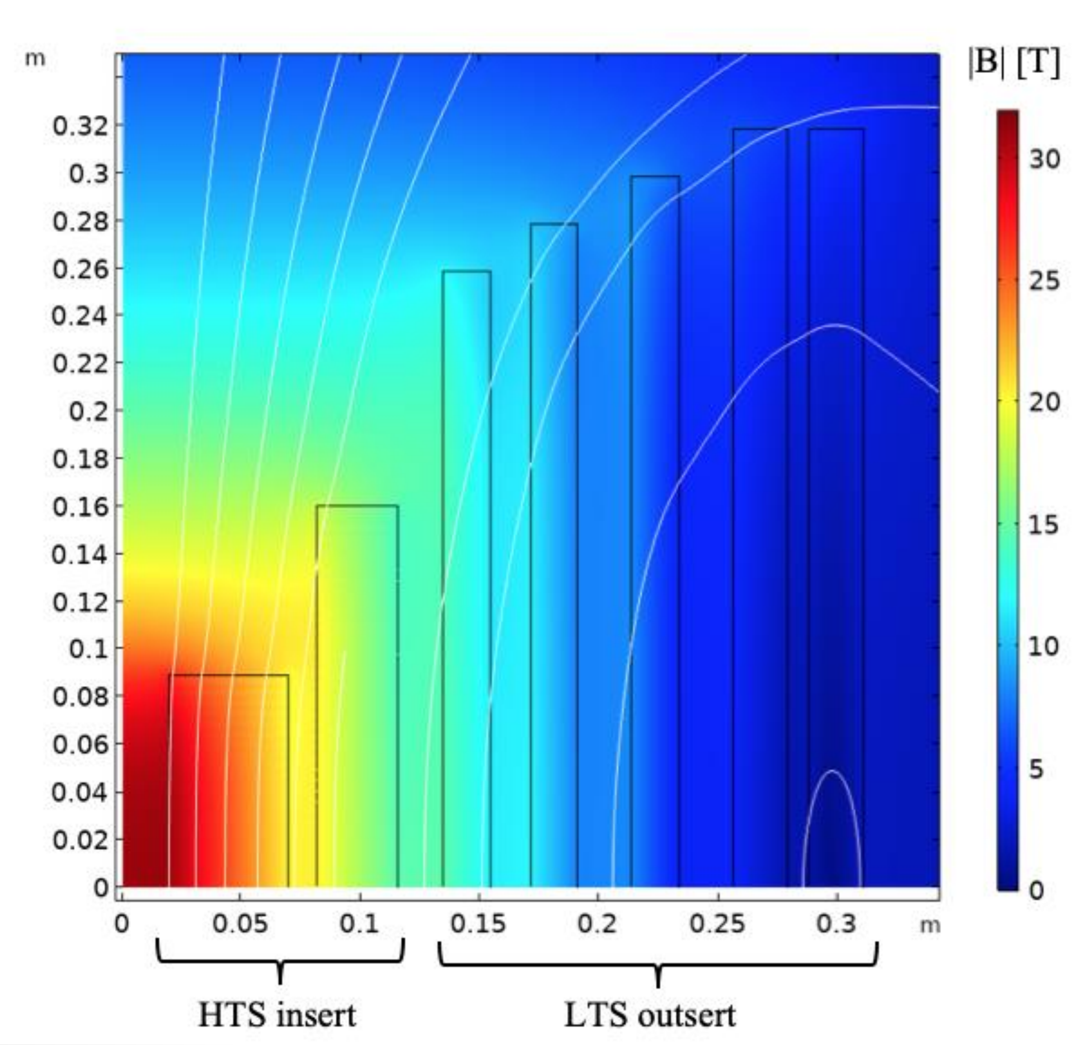
NHMFL 32 T Superconducting Magnet - *T-A* Homogeneous Model

- Both insert and outsert are charged, considering a real charge cycle.

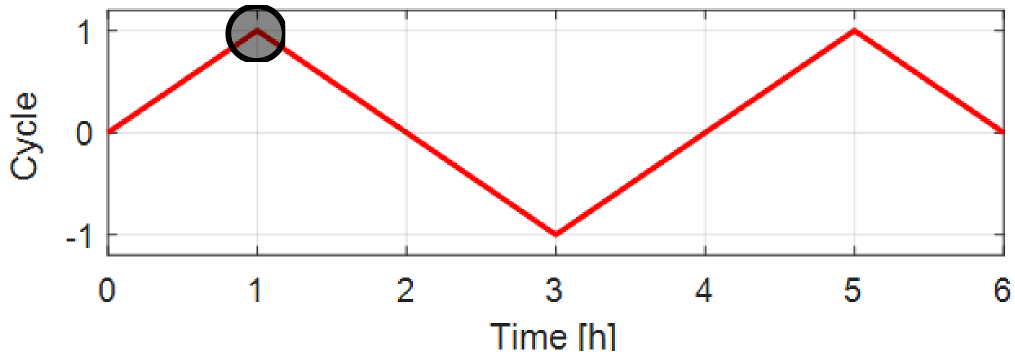


NHMFL 32 T Superconducting Magnet - *T-A* Homogenous Model

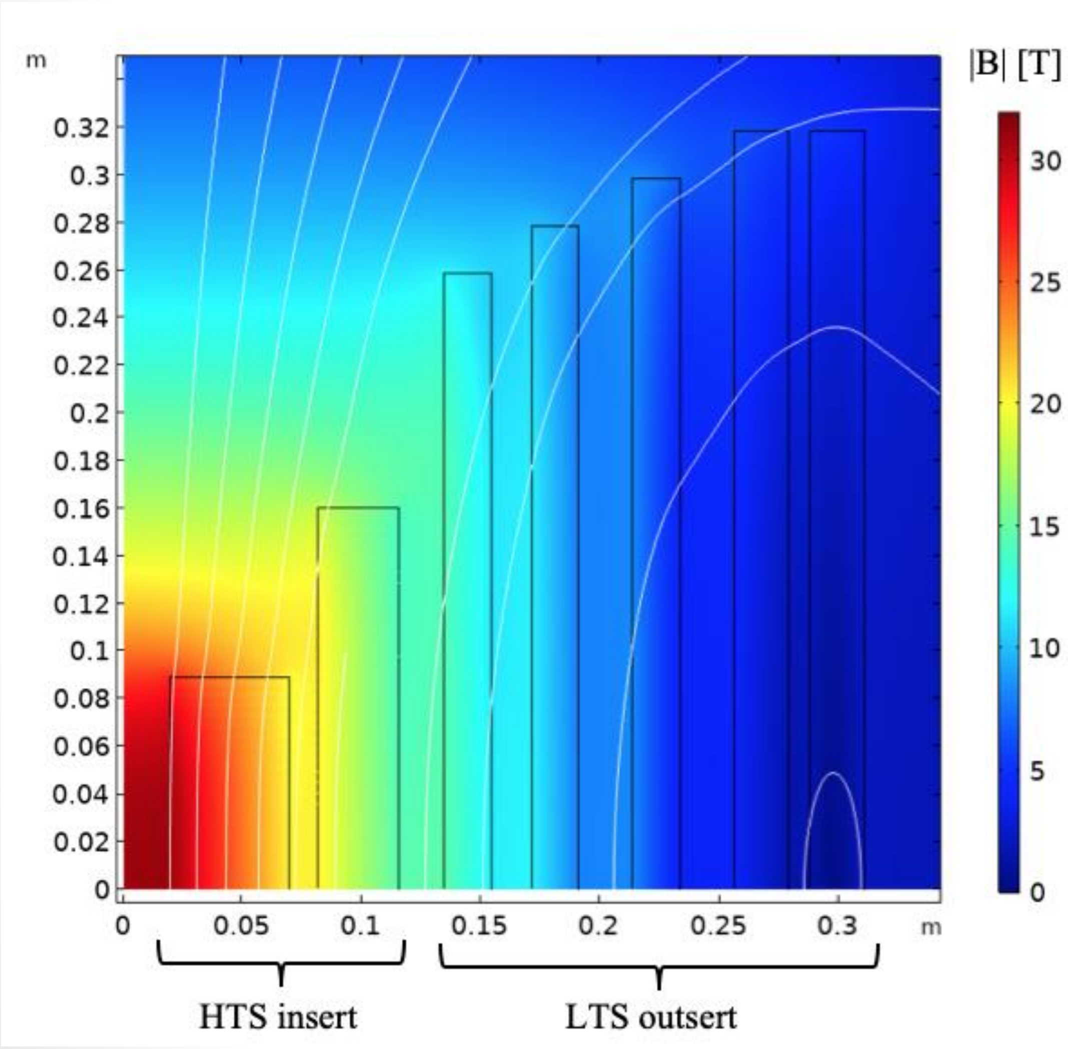
- Both insert and outsert are charged, considering a real charge cycle.



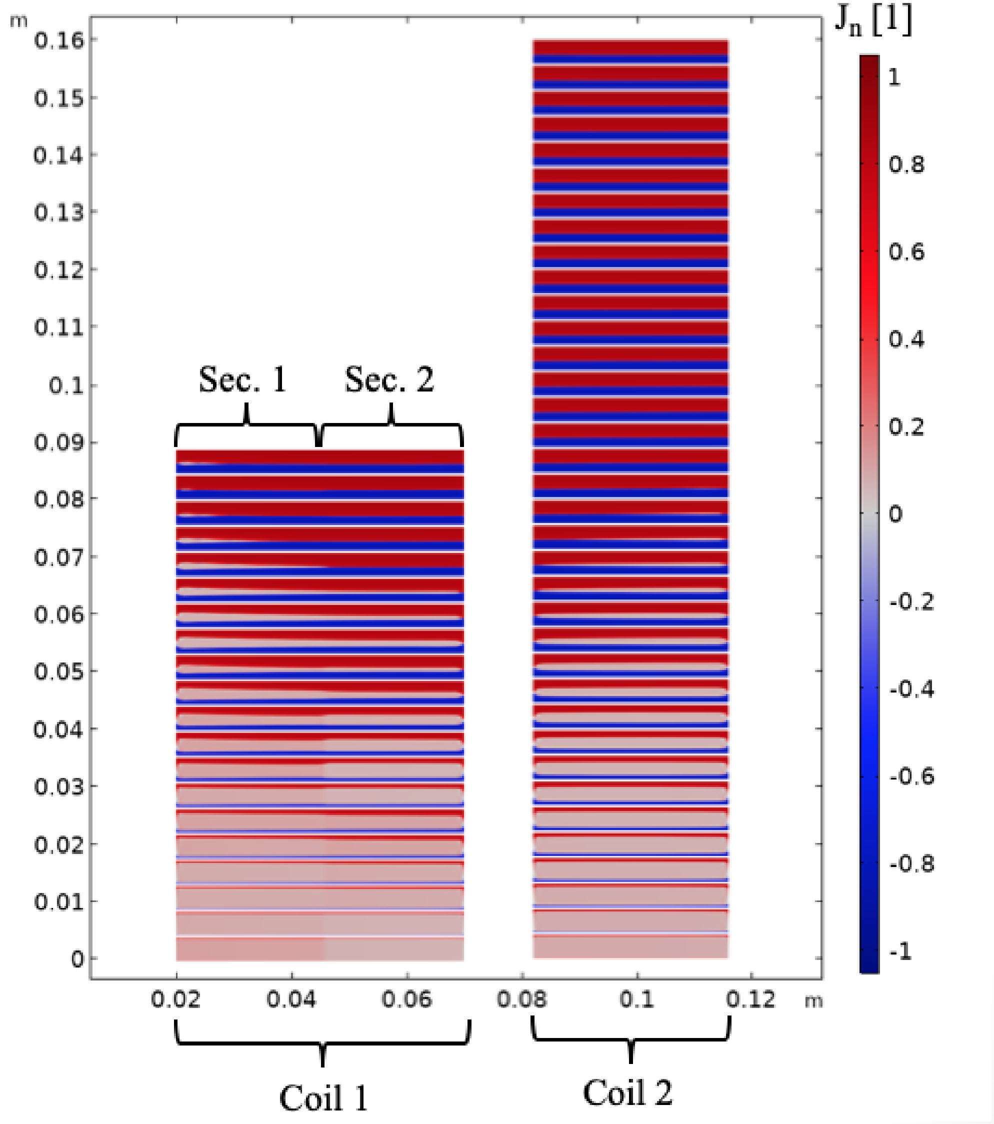
Magnetic field magnitude at peak current.



NHMFL 32 T Superconducting Magnet - *T-A* Homogenous Model

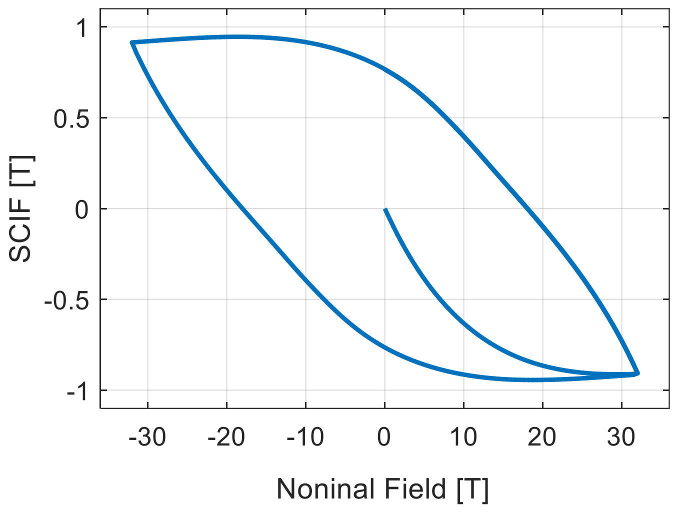


Magnetic field magnitude at peak current.



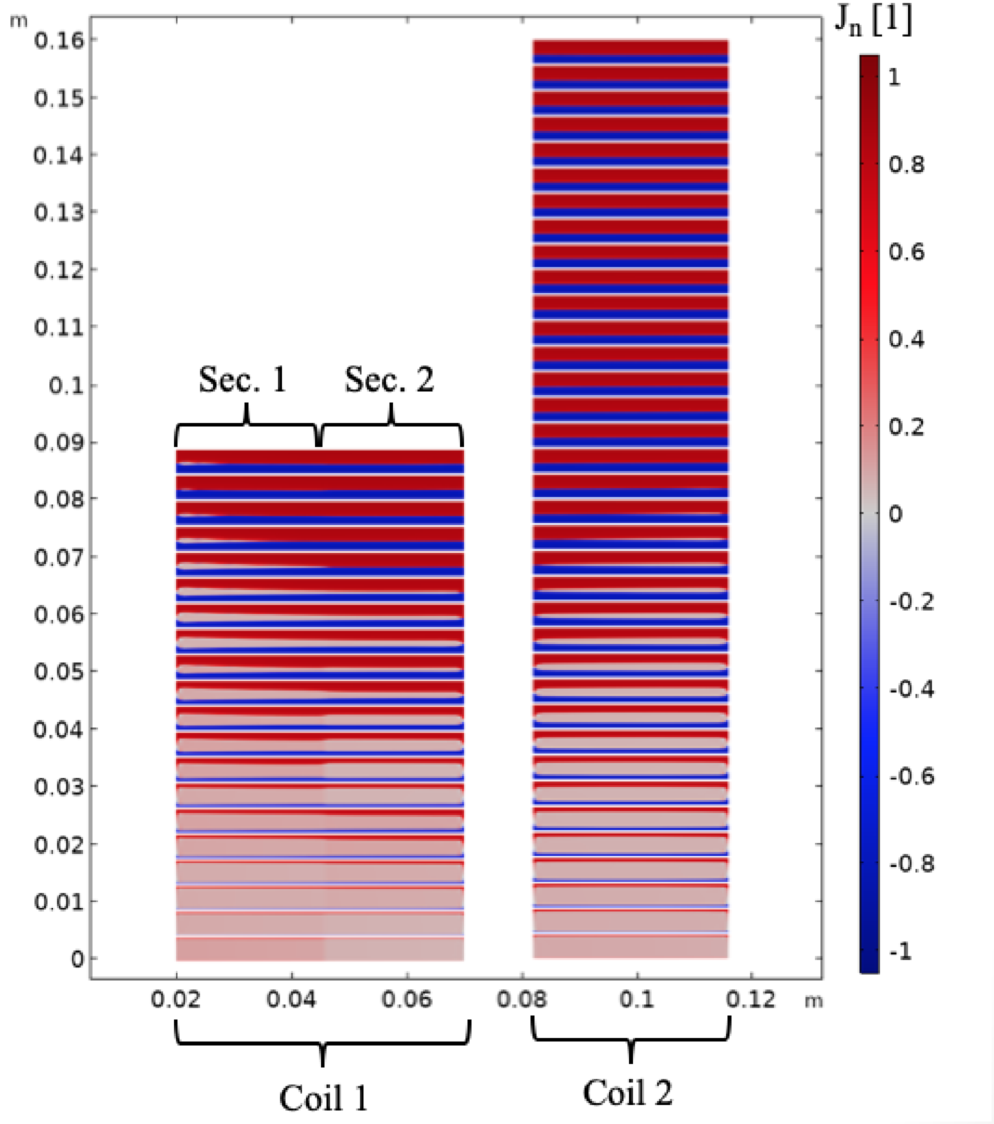
J at peak current in the last iterations.

NHMFL 32 T Superconducting Magnet - *T-A* Homogenous Model



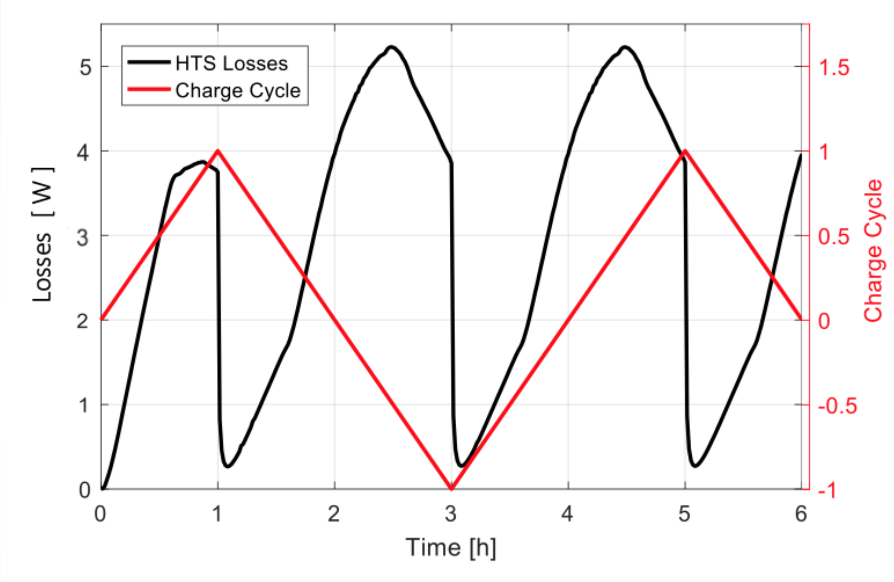
Screening Current Induced Field Loop.

The J distribution is used to compute the stresses, Kolb-Bond *et al. Mon-Af-Po1.11-05: Stress analysis of the 32 T superconducting magnet at the MagLab including screening current effects* [16].

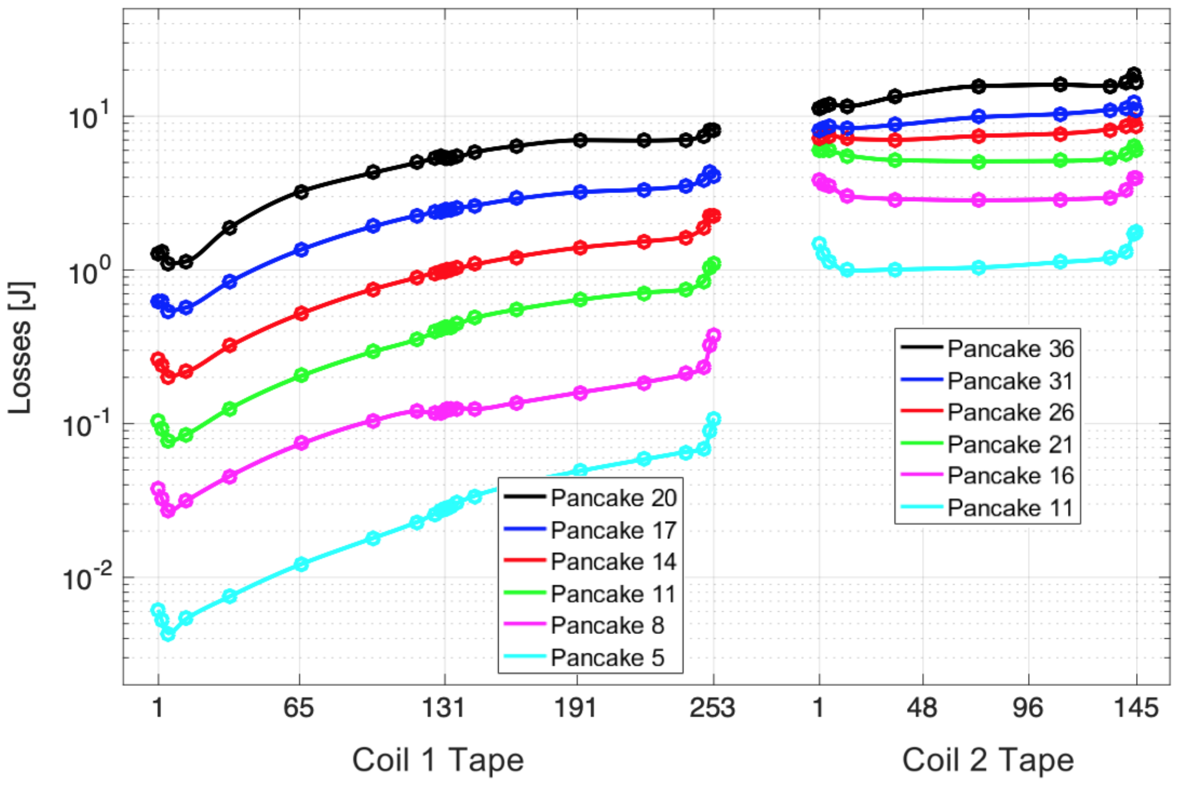


J at peak current in the last iterations.

NHMFL 32 T Superconducting Magnet - *T-A* Homogenous Model

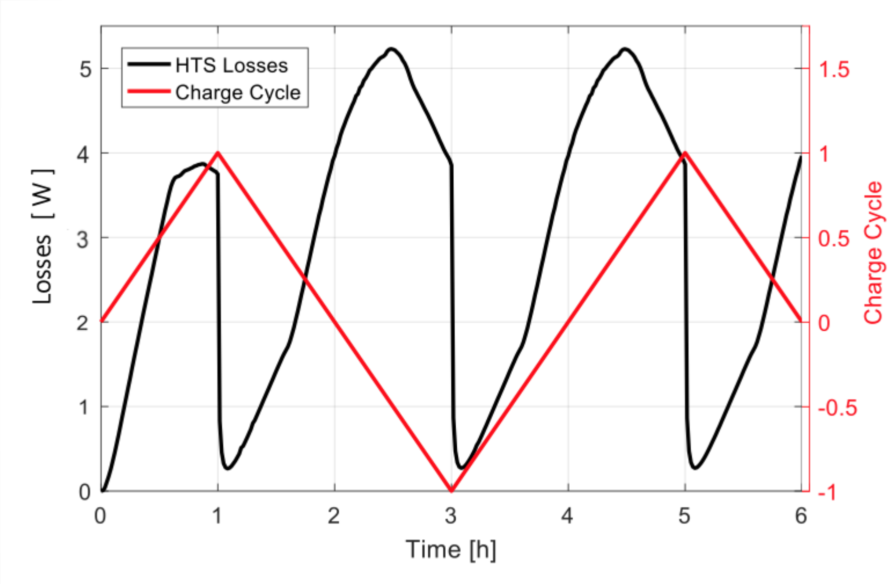


Instantaneous losses and charge cycle

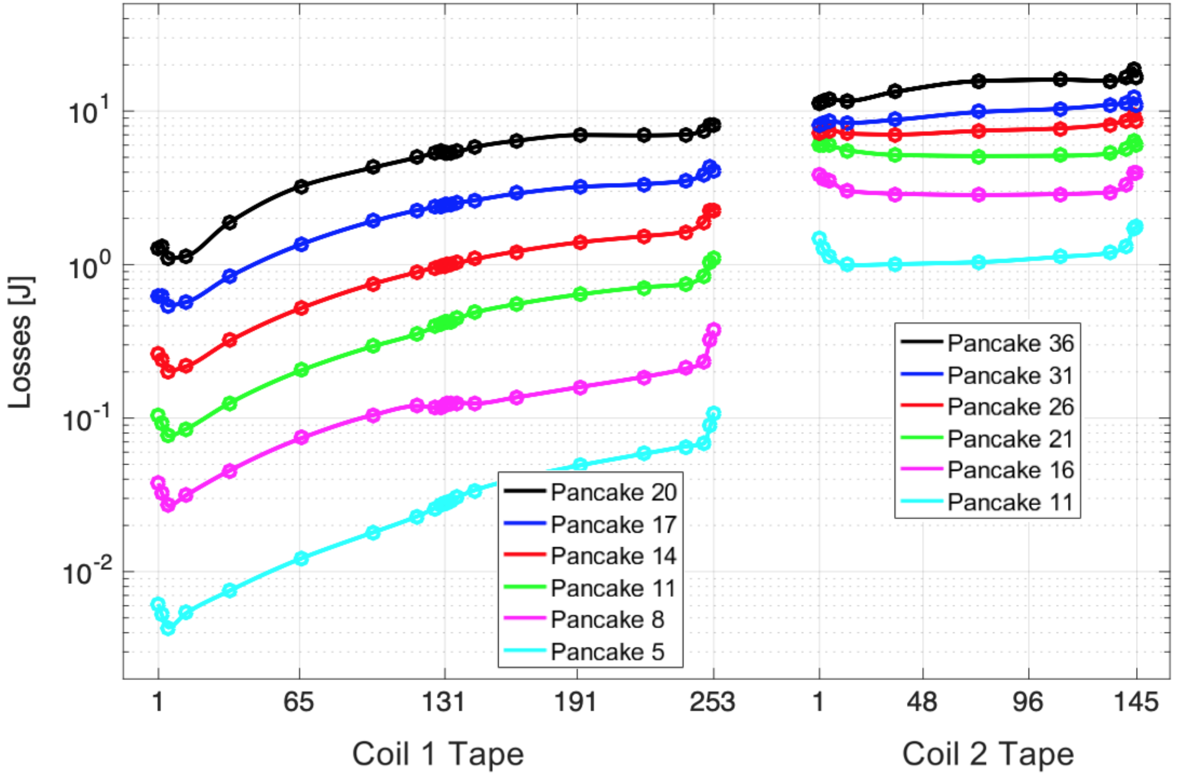


Losses in selected pancakes

NHMFL 32 T Superconducting Magnet - *T-A* Homogenous Model



Instantaneous losses and charge cycle

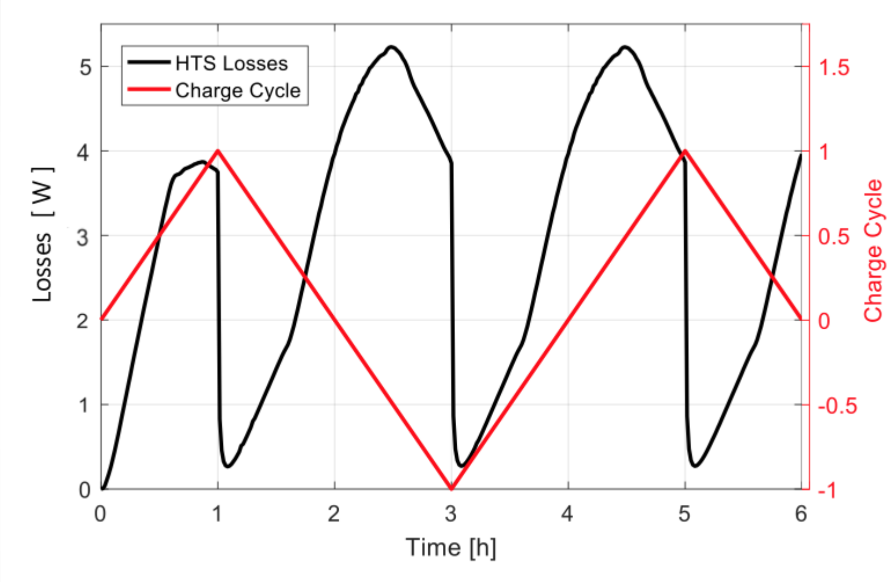


Losses in selected pancakes

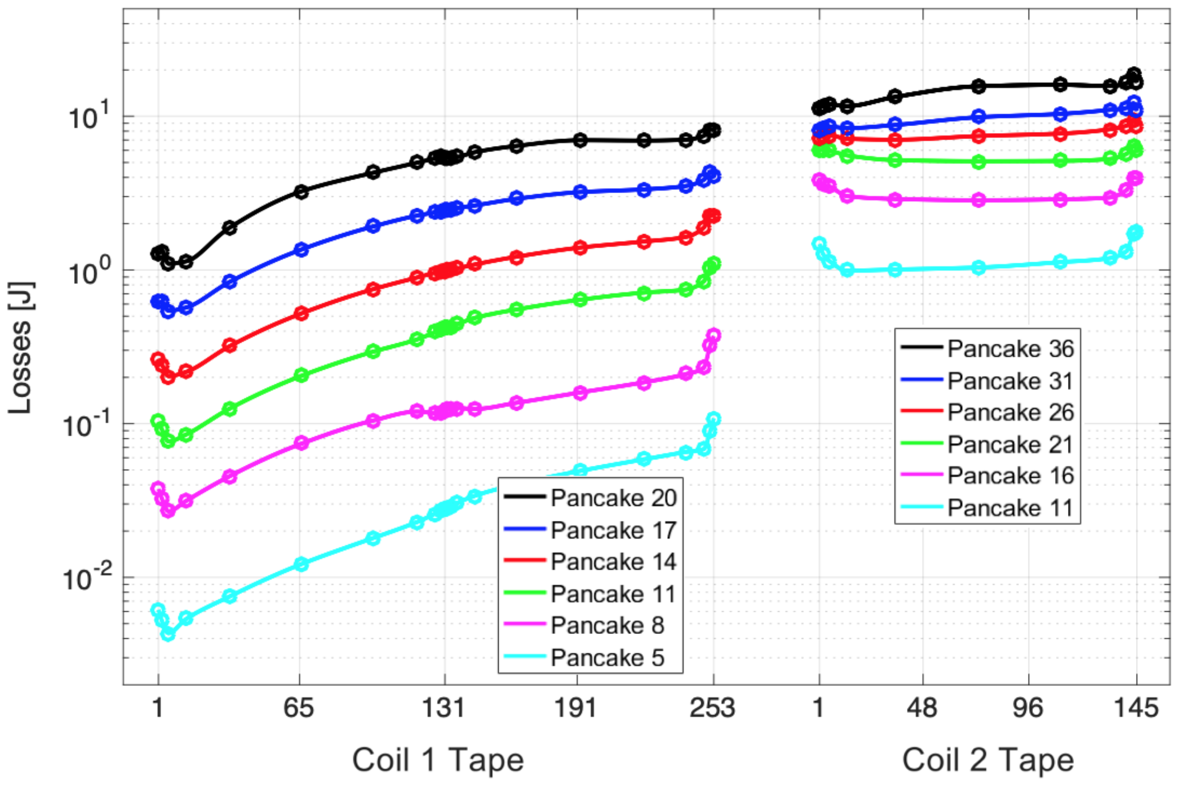
Computation time

- Multi-scale **19 days** (without the LTS outsert field).
- Homogeneous **4 h 15 min.**

NHMFL 32 T Superconducting Magnet - *T-A* Homogenous Model



Instantaneous losses and charge cycle



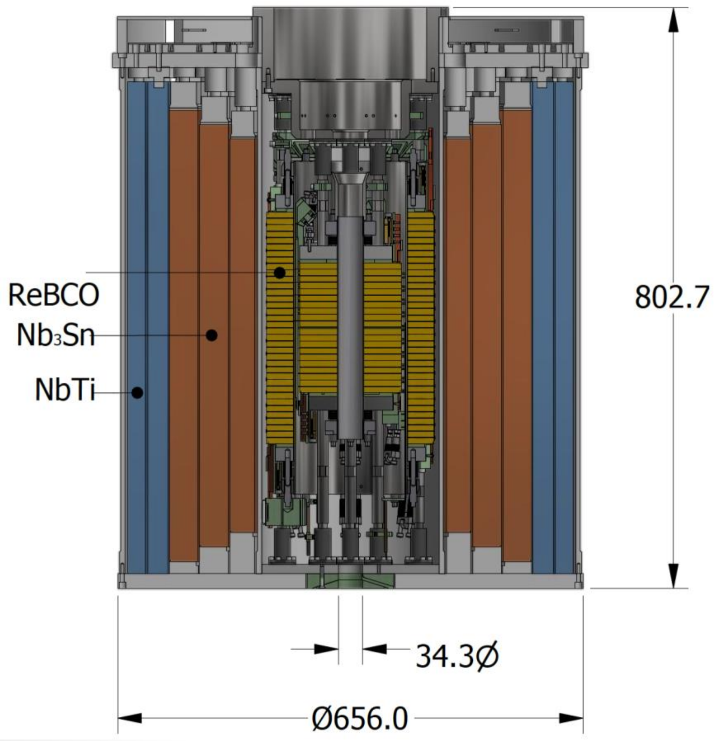
Losses in selected pancakes

Computation time

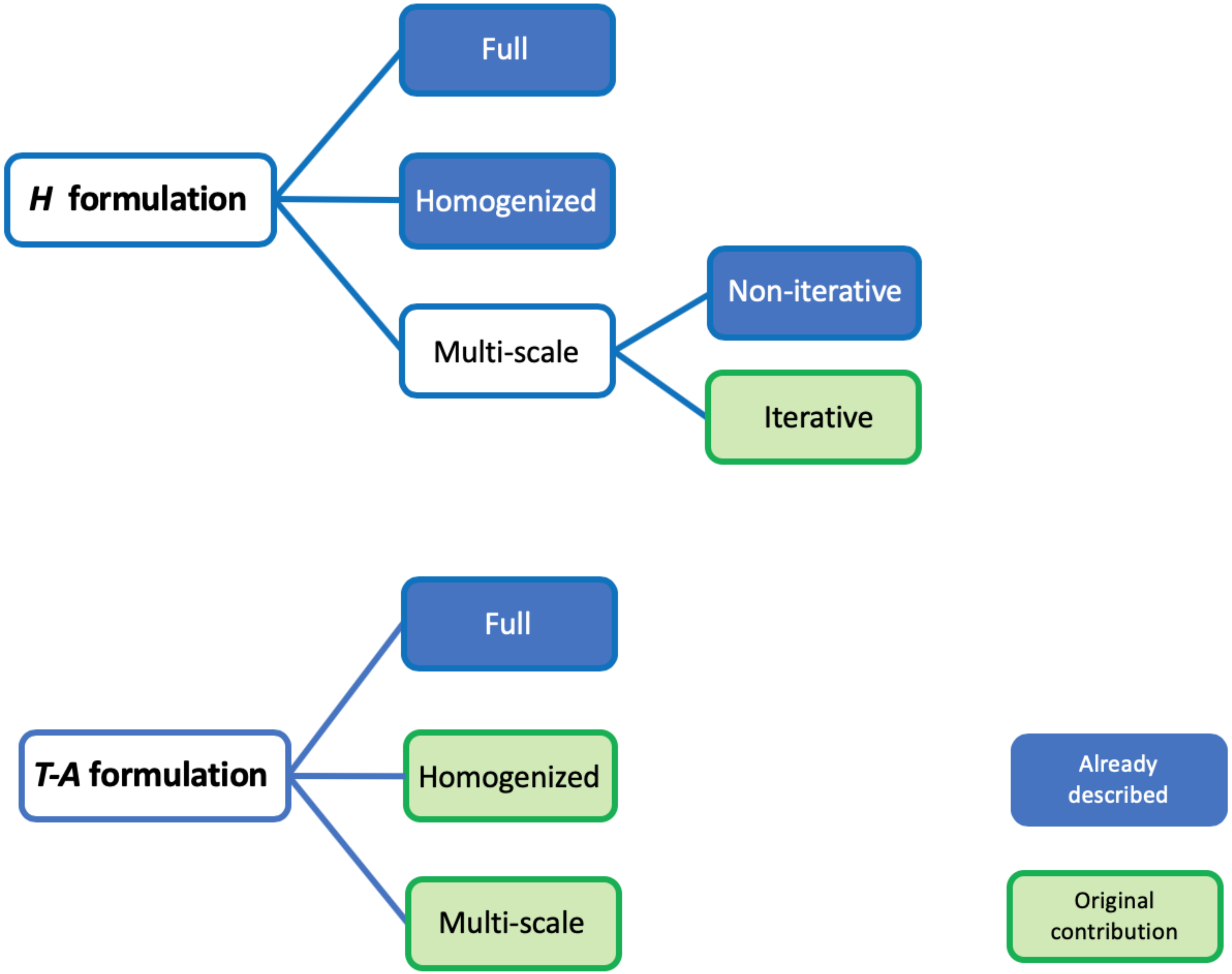
- Multi-scale **19 days** (without the LTS outsert field).
- Homogeneous **4 h 15 min.** → Real-time computation

NHMFL 32 T Superconducting Magnet

NATIONAL HIGH MAGNETIC FIELD LABORATORY



Conclusions



Thank you very much!

References

[1] C. P. Bean (1962). "Magnetization of hard superconductors," Phys. Rev. Lett., 1962

[2] Norris, W. (1970). Calculation of hysteresis losses in hard superconductors carrying ac: isolated conductors and edges of thin sheets. *Journal of Physics D: Applied Physics*, 3(4), pp.489-507.

[3] Brandt, E. (1994). Thin superconductors in a perpendicular magnetic ac field: General formulation and strip geometry. *Physical Review B*, 49(13), pp.9024-9040.

[4] Mawatari, Y. (1996). Critical state of periodically arranged superconducting-strip lines in perpendicular fields. *Physical Review B,* 54(18), pp.13215-13221.

[5] Clem, J. (2008). Field and current distributions and ac losses in a bifilar stack of superconducting strips. *Physical Review B*, 77(13).

[6] Brambilla, R., Grilli, F. and Martini, L. (2006). "Development of an edge-element model for AC loss computation of high-temperature superconductors". Superconductor Science and Technology, 20(1), pp.16-24.

[7] Sotelo, G., Carrera, M., Lopez-Lopez, J. and Granados, X. (2016). H-Formulation FEM Modeling of the Current Distribution in 2G HTS Tapes and Its Experimental Validation Using Hall Probe Mapping. *IEEE Transactions on Applied Superconductivity*, 26(8), pp.1-10

[8] Zermeno, V., Abrahamsen, A., Mijatovic, N., Jensen, B. and Sørensen, M. (2013). Calculation of alternating current losses in stacks and coils made of second generation high temperature superconducting tapes for large scale applications. *Journal of Applied Physics*, 114(17), p.173901.

[9] Loïc Quéval, Víctor M R Zermeño and Francesco Grilli. "Numerical models for ac loss calculation in large-scale applications of HTS coated conductors", *Supercond. Sci. Technol*. 29, 2016.

References

[10] Jing Xia; Bai, H.; Lu, J.;Gavrilin, A.V.; Zhou, Y.; Weijers H. "Electromagnetic modeling of REBCO high field coils by H-formulation" *Supercond. Sci. Technol* vol 28, n 12, Oct 2015.

[11] E. Berrospe-Juarez, V. Zermeño, F. Trillaud and F. Grilli, "Iterative multi-scale method for estimation of hysteresis losses and current density in large-scale HTS systems", *Superconductor Science and Technology*, vol. 31, no. 9, p. 095002, 2018.

[12] E. Berrospe-Juarez et al., "Estimation of Losses in the (RE)BCO Two-Coil Insert of the NHMFL 32 T All-Superconducting Magnet", *IEEE Transactions on Applied Superconductivity*, vol. 28, no. 3, pp. 1-5, 2018.

[13] Zhang, H., Zhang, M. and Yuan, W. (2016). An efficient 3D finite element method model based on the T–A formulation for superconducting coated conductors. *Superconductor Science and Technology*, 30(2), p.024005.

[14] Liang, F., Venuturumilli, S., Zhang, H., Zhang, M., Kvitkovic, J., Pamidi, S., Wang, Y. and Yuan, W. (2017). A finite element model for simulating second generation high temperature superconducting coils/stacks with large number of turns. *Journal of Applied Physics*, 122(4), p.043903.

[15] E. Berrospe-Juarez, V. Zermeño, F. Trillaud and F. Grilli, "Real-time simulation of large-scale HTS systems: multi-scale and homogeneous models using the T–A formulation", *Superconductor Science and Technology*, vol. 32, no. 6, p. 065003, 2019. Available: 10.1088/1361-6668/ab0d66.

[16] D. Kolb-Bond, *et al.*, "Computation of Strains due to Screening Currents in REBCO Magnets at the NHMFL", presented, MT-26, Mon-Af-Po1.11-05, Vancouver, BC, Canada, Sept. 23 – 27, 2019.

Definitions

The average hysteresis losses are obtained using data of the second half of the cycle, as follows,

$$Q_{av} = \frac{2}{P}\int_{P/2}^{P}\int_{\Omega_{sc}} \mathbf{E}\cdot\mathbf{J}\, dS\, dt,$$

where P is the period of the sinusoidal cycle, and Ω_{sc} are the superconducting subdomains.

The average losses relative error, expressed in percent, is defined as,

$$er_Q = \frac{(Q_{M\_av} - Q_{R\_av})}{Q_{R\_av}} \times 100\,\%,$$

where Q_{R_av} and Q_{M_av} are the average losses computed with the reference and with the model that is being compared, respectively.

The J distributions are multivariable functions. The coefficient of determination is defined as,

$$R^2 = 1 - \frac{\sum_{i=1}^{m}(J_R - J_M)^2}{\sum_{i=1}^{m}(J_R - \bar{J}_M)^2},$$

where J_R and J_M are vectors built by concatenating the J distribution of all the tapes for all the time steps, computed with the reference model and with the tested model, respectively. $\bar{J}_R$ is the mean value of J_R. It must be remembered that $R^2 = 1$ means a perfect matching between J_R and J_M.

The normalized computation time is defined as,

$$\bar{ct} = \frac{ct_R}{ct_M},$$

where ct_R and ct_M are the computation time required by the reference model and by the tested model, respectively.

Inverse Cumulative Distribution Function Interpolation